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Data transmission with variable-redundancy error control over a high-frequency channel

R.M.F. Goodman, B.Sc., and P.G. Farrell, B.Sc., Ph.D.

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Abstract

Results of computations and field tests on a binary-data-transmission system, operating at 1 kbaud over an h.f. channel, are presented. Error correction is effected by means of error detection and automatic request for repeat, via a feedback channel (a Post Office private line). A set of short, fixed-block-length cyclic codes is available, a code of appropriate redundancy being automatically selected to match the varying channel conditions. The decision about which code to use is made at the receiver, and the transmitter is informed via the feedback channel. The results show that relatively simple, reliable, and efficient data communication can be realised by this means.

List of symbols

b = burst length
g = guard space (error-free digits between bursts)
k = number of information digits in block
m = number of consecutive errors
n = block length
E = transmission efficiency (data rate)
S = sink-bit-error rate
U = sink-block-error rate
a.r.q. = automatic request for repeat
e.d. = error detection
f.e.c. = forward-error control
f.s.k. = frequency-shift keying
p.n. = pseudonoise

1 Introduction

The uncoded error rate of many practical digital-data-transmission links varies with time, due to fading, multipath and other effects; consequently, if the system is to be reliable, the error-control code used must be powerful enough to cope with the worst error conditions. Since a powerful code is usually either complex or inefficient, or both, the system is unneccessarily complex or inefficient when operating at average or low error rates (an inefficient code means a code with a low effective data rate, in the context of this paper).

A possible solution of the problem is to use a set of codes, of varying redundancy, instead of just one powerful code. The set of codes is chosen to cover the range of error conditions expected, but now only the codes for low or average error rate need to be efficient (although all should be optimum), since the link will probably operate in this condition for most of the time. The receiving terminal of the link must be capable of estimating the link error rate, and a feedback link is required to pass the estimate to the encoder, so that a code with appropriate redundancy can be selected. The presence of the feedback link also means that a retransmission method of coding, such as error detection with automatic request for a repeat (e.d.–a.r.q.), or hybrid forward-error correction (f.e.c.) and e.d.–a.r.q., can be used, with a consequent increase of efficiency or decrease in complexity.

The combined advantages of variable-redundancy coding and a.r.q. (or f.e.c.–a.r.q.) mean that it is feasible to consider the use of sets of relatively short (block length $n \leqslant 50$) optimum or nearly optimum block codes. In practice, it is best to use a set of codes all having the same block length. The results of several simulations show that simple, reliable and efficient links can be realised by using the variable-redundancy technique.[1]

This paper presents the results of computations and field tests on a practical binary-data-transmission system with variable-redundancy error-control coding, operating between West Wellow (Hants.) and Canterbury (Kent) at a baud rate of 1 kbaud, over an h.f. forward channel, and with a Post Office private-line feedback channel.

2 Experimental system

A block diagram of the system is shown in Fig. 1, and a brief specification is given below:

carrier frequency	= 7·375 MHz
modulation	= binary f.s.k.: 7·375 MHz ± 425 Hz
baud rate	= 1 kbaud

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Dr. Farrell is, and Mr. Goodman was formerly, with the Electronics Laboratories, University of Kent, Canterbury, Kent CT2 7NT, England. Mr. Goodman is now with the School of Electrical & Electronic Engineering, Kingston Polytechnic, Kingston upon Thames, Surrey, England

bandwidth = 3 kHz
receiver and transmitter aerials = halfwave dipoles
h.f. path length ≃ 190 km (120 miles)(single-hop main path)
radiated power ≃ 75 W
received signal strength ≃ 0·15 μV/m
feedback signalling = two audio tones
error-control coding = binary cyclic codes with block length $n = 15$, and number of information digits k given by $14 \geqslant k \geqslant 1$ (see Table 1).

Table 1

CYCLIC-CODE PARAMETERS

Code (n, k)	Efficiency (data rate) (k/n)	Hamming distance
	%	
(15, 14)	93	2
(15, 11)	73	3
(15, 10)	67	4
(15, 7)	47	5
(15, 6)	40	6
(15, 4)	27	8
(15, 2)	13·3	9
(15, 1)	6·7	15

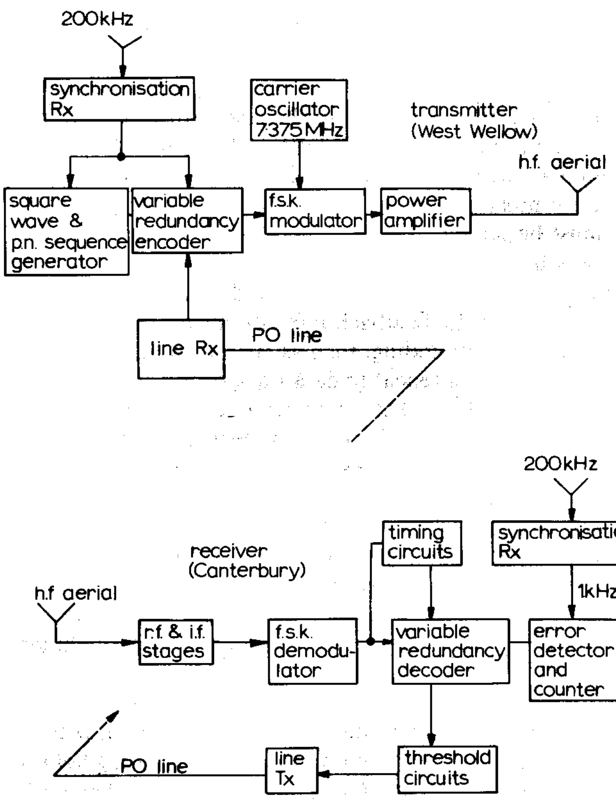

Fig. 1
Experimental system

The timing synchronisation for the 1 kbaud rate at West Wellow and the error counter at Canterbury was derived from an Airmec receiver locked to the 200 kHz Droitwich transmission. The GPO wire could not be used for this purpose, because the s.s.b. circuits involved do not have locked carriers.

The f.s.k. detector was constructed from three active-frequency selective devices (digital discriminators or Z-trip circuits): two were tuned to respond to the mark and space frequencies, respectively, and a third was tuned to indicate whether the signal was above or below the carrier frequency. The outputs of one of each of the first two Z-trips were then combined with the third in an AND gate to give a reliable indication of mark or space. This relatively novel type of detector was found to have a good performance.

Further details of the circuits, codes and operating methods of the system will be available in Reference 2.

3 Results

3.1 Propagation conditions

The number of frequencies available for the tests was limited, since complications in the transmitter and receiver circuitry had to be avoided as far as possible, and automatic operation with one person monitoring the system in Canterbury was desirable. Thus, after initial tests, and consultation on frequency predictions, it was decided to fix the frequency at 7·375 MHz.

Table 2

BIT-ERROR RATES (TRANSMITTING REVERSALS)

Test number	Duration, min	Bit-error rate	Average received power, dB above 1 μV	Fading rate across average /min	Fading rate across 10 dB down from average/ min (e)
1*a*	37	$1{\cdot}66 \times 10^{-3}$	27	0·46	0·2
2*b*	33	$6{\cdot}7 \times 10^{-6}$	31	0·06	0
3*b*	31	$2{\cdot}1 \times 10^{-5}$	31	0·03	0
4*c*	30	$1{\cdot}1 \times 10^{-3}$	30	0·26	0·167
5	39	$9{\cdot}7 \times 10^{-3}$	43	0·3	0
6	10	$1{\cdot}4 \times 10^{-4}$	33	0·2	0
7	10	$2{\cdot}33 \times 10^{-2}$	33	0·2	0
8	10	$9{\cdot}0 \times 10^{-4}$	26	0·6	0·3
9	11	$4{\cdot}9 \times 10^{-3}$	26	5·2	1·5
10*d*	11	$8{\cdot}0 \times 10^{-3}$	27	11·8	2·45
11*d*	10	$1{\cdot}1 \times 10^{-2}$	29	4·7	3·1
12	12	$1{\cdot}52 \times 10^{-3}$	36	3·58	1·16
13	12	$6{\cdot}8 \times 10^{-6}$	32	0·46	0·17
14	10	$1{\cdot}8 \times 10^{-4}$	32	0·1	0·1
15	11	$2{\cdot}1 \times 10^{-3}$	28	2·0	0·27
16	11	$5{\cdot}4 \times 10^{-3}$	28	5·8	2·6

a flat fades
b steady received power
c some voice interference
d selective fading
e gives an indication of very deep fading

Night reception (7 p.m.–2 a.m.) at this frequency was normally impossible due to strong broadcasting interference. During the early hours of the morning (2–7 a.m.), the maximum usable frequency (m.u.f.) limited propagation. During the day, reception was adequate, except for frequency-selective fading (0·5–60 s duration) and interference from ground-to-air communications and high-speed teleprinter traffic. The usable signal time (bit probability of error $< 10^{-1}$) averaged out at about 30% of the 24 h during the tests. The lowest usable frequency (l.u.f.) did not affect the tests.

Multipath effects were observed, as expected. Amplitude-modulation soundings showed strong echoes with about 1 ms delay, and weaker echoes with up to about 5 ms delay. The echo-fading periods were between 0·5 and 5·0 s.

3.2 Error rates without coding

3.2.1 Transmitting reversals

Table 2 shows bit-error rates for several tests. The lowest error rate (apart from two error-free tests) is $6{\cdot}7 \times 10^{-6}$, the highest (excluding all tests with more than one error in ten digits) is $2{\cdot}33 \times 10^{-2}$. The average error rate over all usable tests is very approximately 1×10^{-3}. The correlations between received-signal power and fading rate, and error rate, are poor; even though the selected error-counting periods were chosen to be relatively free of interference (voice, etc.), so that flat or frequency-selective fading, or multipath, were the main causes of errors.

3.2.2 Transmitting a pseudonoise sequence (length 15 digits)

21 tests, each lasting for approximately three-quarters of an hour were recorded, and 11 of these were selected for analysis (see Table 3). The error statistics obtained were used to model three random-error and eleven burst-error channels to provide further data for computer calculations. The details for the selected tests are:

Table 3

BIT-ERROR RATES AND BLOCK-ERROR RATES (TRANSMITTING 15-BIT P.N. SEQUENCE)

Test	Bit-error rates	Block-error rates ($n = 15$)
17	$6{\cdot}23 \times 10^{-3}$	$3{\cdot}5 \times 10^{-2}$
18	$8{\cdot}02 \times 10^{-3}$	$3{\cdot}7 \times 10^{-2}$
19	$1{\cdot}93 \times 10^{-2}$	6×10^{-2}
20	$1{\cdot}01 \times 10^{-2}$	$6{\cdot}9 \times 10^{-2}$
21	$1{\cdot}66 \times 10^{-2}$	$7{\cdot}7 \times 10^{-2}$
22	$2{\cdot}15 \times 10^{-2}$	0·1
23	$1{\cdot}87 \times 10^{-2}$	0·1
24	$3{\cdot}47 \times 10^{-2}$	0·12
25	$3{\cdot}62 \times 10^{-2}$	0·16
26	$5{\cdot}63 \times 10^{-2}$	0·17
27	$5{\cdot}38 \times 10^{-2}$	0·28

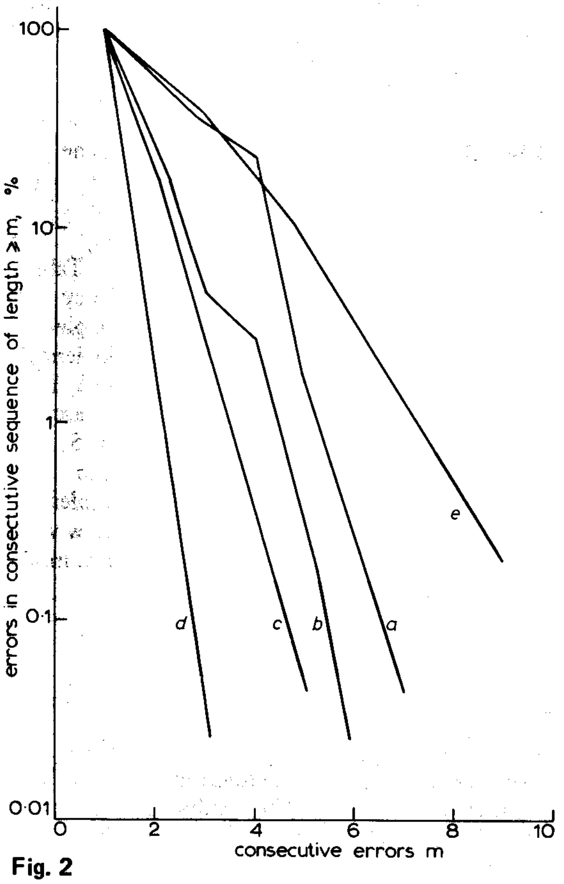

Fig. 2
Percentage of errors in a consecutive-error sequence of length ≥ m, plotted against m

a Test 17 (lowest error rate)
b Test 19
c Binary symmetric channel model, $p = 10^{-1}$
d Binary symmetric channel model, $p = 10^{-2}$
e Burst channel model, $p = 6{\cdot}8 \times 10^{-3}$

lowest bit-error rate = $6{\cdot}2 \times 10^{-3}$ (1 in 161)
average bit-error rate = $2{\cdot}5 \times 10^{-2}$ (1 in 40)
highest bit-error rate = $5{\cdot}6 \times 10^{-2}$ (1 in 18)

Errors occured in bursts of length $b \leq 300$ with up to 10^4 error-free digits in between, see Figs. 2 and 3.

Periods with high proportions of short bursts were those affected by multipath. The average error rates are higher, compared with those for reversals transmission, mainly because of the random nature of the signal and the consequently wider frequency spectrum involved.

3.3 Performance of system with coding: computed results

The 11 test recordings and 14 models mentioned in Section 3.2.2 were processed to provide digital channel statistics for several computer-simulated error-control systems. Sink-(output) error rates and transmission efficiencies were computed for the various systems; some of the results are given below.

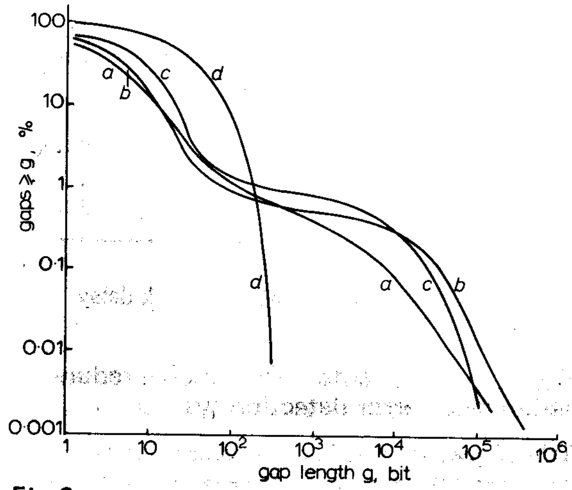

Fig. 3
Percentage of error-free gaps ≥ g, against g

a All 11 tests (17–21)
b Test 18
c Test 19
d Binary symmetric channel model with error rate equal to average error rate for all 11 tests

3,3.1 Error detection using fixed-redundancy cyclic code with $n = 15$

Table 4 gives the computed sink-block-error rate for all eight cyclic codes, ranging from the ($n = 15$, $k = 14$) code to the (15, 1) code. Also shown is the effect on the results of interleaving to degrees 2 and 3; a general improvement is noted. Interleaving to higher degrees (up to 10) did not give correspondingly better results, because of multipath affecting many successive blocks.

3.3.2 Error detection using 2-code variable-redundancy scheme

Two sets of codes were used, namely (15, 14) (15, 1) and (15, 11) (15, 1). The scheme consists of changing to the more powerful code when an erroneous block is detected, and back to the less powerful (but higher data rate) code when a block does not contain errors. One block delay is allowed before making the change. Table 5 shows the efficiency of transmission E and the sink-block-error rate U, for the two code combinations. Interleaving to degree 2 is not useful: E is increased a little, but U also increases slightly.

3.3.3 Error correction by e.d.–a.r.q. with fixed and variable redundancy

Results are given in Table 6. In almost every case, the use of variable redundancy improves the sink-bit-error

Table 4
COMPUTED PERFORMANCE OF CODES: SINK-BLOCK-ERROR RATES

Test	Code							
	(15, 14)	(15, 11)	(15, 10)	(15, 7)	(15, 6)	(15, 4)	(15, 2)	(15, 1)
17	$1{\cdot}6 \times 10^{-2}$	$5{\cdot}8 \times 10^{-4}$	$2{\cdot}1 \times 10^{-4}$	$5{\cdot}4 \times 10^{-5}$	$5{\cdot}4 \times 10^{-6}$	0	0	0
18	$1{\cdot}6 \times 10^{-2}$	$3{\cdot}5 \times 10^{-3}$	$3{\cdot}4 \times 10^{-3}$	$3{\cdot}2 \times 10^{-3}$	$3{\cdot}2 \times 10^{-3}$	$3{\cdot}2 \times 10^{-3}$	0	0
19	$4{\cdot}0 \times 10^{-2}$	$9{\cdot}5 \times 10^{-3}$	$9{\cdot}5 \times 10^{-3}$	$9{\cdot}1 \times 10^{-3}$	$9{\cdot}1 \times 10^{-3}$	$9{\cdot}1 \times 10^{-3}$	0	0
20	$2{\cdot}5 \times 10^{-2}$	$3{\cdot}4 \times 10^{-4}$	$2{\cdot}2 \times 10^{-4}$	0	0	0	0	0
21	$3{\cdot}5 \times 10^{-2}$	$2{\cdot}6 \times 10^{-3}$	$2{\cdot}1 \times 10^{-3}$	$1{\cdot}7 \times 10^{-3}$	$1{\cdot}6 \times 10^{-3}$	$1{\cdot}6 \times 10^{-3}$	0	0
22	$4{\cdot}9 \times 10^{-2}$	4×10^{-3}	$3{\cdot}7 \times 10^{-3}$	$3{\cdot}6 \times 10^{-4}$	$3{\cdot}5 \times 10^{-4}$	$3{\cdot}5 \times 10^{-4}$	0	0
23	4×10^{-2}	$3{\cdot}4 \times 10^{-3}$	$2{\cdot}4 \times 10^{-3}$	$7{\cdot}4 \times 10^{-5}$	$1{\cdot}7 \times 10^{-5}$	$1{\cdot}1 \times 10^{-5}$	0	0
24	$5{\cdot}7 \times 10^{-2}$	$2{\cdot}9 \times 10^{-2}$	$2{\cdot}2 \times 10^{-2}$	$2{\cdot}1 \times 10^{-2}$	$1{\cdot}5 \times 10^{-2}$	$1{\cdot}5 \times 10^{-2}$	0	0
25	$7{\cdot}5 \times 10^{-2}$	$2{\cdot}2 \times 10^{-2}$	$2{\cdot}1 \times 10^{-2}$	$4{\cdot}2 \times 10^{-3}$	$4{\cdot}2 \times 10^{-3}$	$4{\cdot}1 \times 10^{-3}$	0	0
26	$9{\cdot}8 \times 10^{-2}$	5×10^{-2}	$4{\cdot}9 \times 10^{-2}$	$4{\cdot}5 \times 10^{-2}$	$4{\cdot}5 \times 10^{-2}$	$4{\cdot}5 \times 10^{-2}$	0	0
27	1×10^{-1}	$1{\cdot}1 \times 10^{-2}$	$3{\cdot}4 \times 10^{-3}$	$3{\cdot}6 \times 10^{-3}$	$1{\cdot}7 \times 10^{-4}$	1×10^{-4}	0	0
Interleaving degree 2								
17	$1{\cdot}4 \times 10^{-2}$	$8{\cdot}0 \times 10^{-4}$	$3{\cdot}5 \times 10^{-4}$	0	0	0	0	0
22	$3{\cdot}7 \times 10^{-2}$	$1{\cdot}2 \times 10^{-3}$	$7{\cdot}1 \times 10^{-4}$	$4{\cdot}1 \times 10^{-4}$	$4{\cdot}1 \times 10^{-4}$	0	0	0
26	$5{\cdot}6 \times 10^{-2}$	$4{\cdot}4 \times 10^{-3}$	$2{\cdot}3 \times 10^{-3}$	$4{\cdot}3 \times 10^{-5}$	$2{\cdot}1 \times 10^{-5}$	0	0	0
Interleaving degree 3								
17	$1{\cdot}1 \times 10^{-3}$	$5{\cdot}8 \times 10^{-4}$	3×10^{-4}	$5{\cdot}4 \times 10^{-6}$	$5{\cdot}4 \times 10^{-6}$	0	0	0
22	$3{\cdot}6 \times 10^{-2}$	$1{\cdot}7 \times 10^{-3}$	8×10^{-4}	$9{\cdot}7 \times 10^{-5}$	0	0	0	0
26	$9{\cdot}1 \times 10^{-2}$	$2{\cdot}8 \times 10^{-3}$	$1{\cdot}3 \times 10^{-3}$	$5{\cdot}8 \times 10^{-5}$	$1{\cdot}6 \times 10^{-5}$	0	0	0

rate S (i.e. after attempted correction). A 4-block delay was assumed before change of code.

3.4 Performance of automatic variable-redun-redundancy error detection system

Table 7 gives the results of 19 recorded tests with the experimental automatic variable-redundancy system, operating in an error-detection mode without a.r.q. The set of eight codes was available at the transmitter, and the change from a particular code to another (more or less powerful) was initiated by a counter monitoring the block-error rate (a.r.q rate) passing through one of a given set of thresholds. The thresholds could be adjusted between runs by altering a patch-board interconnection matrix. Because of a difficulty with the counter-reset circuitry (if no errors occurred, it took 1000 blocks for a command to change to a less powerful, and therefore more efficient, code), the efficiency values in Table 7 are not as high as expected. Table 8, however, shows the results of processing ten of the runs to indicate the performance of error correction via a.r.q. These efficiencies are much higher because the effect of the reset delay has been cancelled out.

During these runs it was observed that, in practice, relatively few of the available codes were used for any length of time by the variable-redundancy system, so Table 8 also shows results for two 2-code variable-redundancy systems for comparison. Computed results for the single-parity-check (15, 14) code are given as well. The efficiency values are, of course, slightly lower in the (15, 11) (15, 1) case, but are almost identical in the (15, 14) (15, 1) case. However, the error rates are better for the (15, 11) (15, 1) system. Thus, 2-code variable-redundancy systems can offer advantages in performance, as well as being simpler to realise and operate. It is also interesting to note how well the single-parity-check code operates in the e.d.–a.r.q. mode.

Table 5
COMPUTED PERFORMANCE OF 2-CODE E.D. VARIABLE-REDUNDANCY SCHEMES

Test	Codes			
	(15, 14) (15, 1)		(15, 11) (15, 1)	
	U	E	U	E
		%		%
17	4×10^{-3}	90·6	$1{\cdot}6 \times 10^{-5}$	71
18	$4{\cdot}3 \times 10^{-4}$	90·1	0	70·8
19	4×10^{-3}	88·4	$1{\cdot}6 \times 10^{-5}$	69·3
20	$1{\cdot}7 \times 10^{-3}$	87·4	$1{\cdot}7 \times 10^{-5}$	68·7
21	$2{\cdot}6 \times 10^{-3}$	86·8	$1{\cdot}2 \times 10^{-5}$	68·1
22	6×10^{-3}	84·9	$2{\cdot}7 \times 10^{-5}$	66·5
23	$4{\cdot}4 \times 10^{-3}$	84·8	$2{\cdot}8 \times 10^{-5}$	66·5
24	$5{\cdot}8 \times 10^{-3}$	83·4	$1{\cdot}6 \times 10^{-3}$	66·5
25	$2{\cdot}2 \times 10^{-3}$	79·3	$1{\cdot}1 \times 10^{-4}$	62·4
26	$6{\cdot}1 \times 10^{-3}$	78·8	$1{\cdot}2 \times 10^{-4}$	61·8
27	2×10^{-2}	69·4	$5{\cdot}32 \times 10^{-4}$	54
Interleaving degree 2				
7	18	91·2		
11	8·3	85·4		
9	6·4	79·8		

U = Sink-block-error rate
E = Transmission efficiency

4 Discussion and conclusions

The propagation results are as expected. They correspond well with the available predictions, and confirm the work of other investigators.[3] The results emphasise that, in order to avoid multipath effects, which are the most severe form of propagation interference, the carrier frequency must always be kept as close as possible below the m.u.f., particularly if relatively high baud rates are being used.

Severe broadcast and other inband interference was observed, which very often made the link unusable. This is another reason for having a wide choice of frequencies for a practical high-speed narrow-band link.

The error-rate measurements confirm the severity of propagation and interference effects; the error rate is often quite high, and varies rapidly. Synchronisation problems (frame synchronisation in particular) lead to bursts of

Table 6
COMPUTED PERFORMANCE OF VARIABLE-REDUNDANCY SCHEMES WITH A.R.Q.

	Test 17		Test 22		Test 26		Burst model	
Bit-error rate	$6{\cdot}23 \times 10^{-3}$		$2{\cdot}15 \times 10^{-2}$		$5{\cdot}63 \times 10^{-2}$		$6{\cdot}8 \times 10^{-3}$	
Codes	*S S*	*E*	*S*	*E*	*S*	*E*	*S*	*E*
(15, 14)	$1{\cdot}9 \times 10^{-3}$	90	9×10^{-3}	84	$3{\cdot}4 \times 10^{-2}$	81	$1{\cdot}6 \times 10^{-3}$	89
(15, 14) (15, 11)	$5{\cdot}2 \times 10^{-4}$	89	$1{\cdot}2 \times 10^{-3}$	82	$2{\cdot}7 \times 10^{-2}$	77	$1{\cdot}5 \times 10^{-3}$	88
(15, 14) (15, 1)	$1{\cdot}5 \times 10^{-4}$	88·5	5×10^{-4}	80	6×10^{-4}	73	$1{\cdot}5 \times 10^{-3}$	84
(15, 14) (15, 11) (15, 1)	$4{\cdot}4 \times 10^{-4}$	88	$4{\cdot}7 \times 10^{-4}$	80	$6{\cdot}1 \times 10^{-4}$	73	$1{\cdot}5 \times 10^{-3}$	87
8 code	4×10^{-4}	88	$3{\cdot}4 \times 10^{-4}$	78	$5{\cdot}7 \times 10^{-4}$	72	$1{\cdot}5 \times 10^{-3}$	88
(15, 11)	$7{\cdot}9 \times 10^{-5}$	73	$6{\cdot}7 \times 10^{-4}$	73	$2{\cdot}6 \times 10^{-2}$	73	1×10^{-4}	69
(15, 11) (15, 10)	$1{\cdot}8 \times 10^{-5}$	70	$5{\cdot}7 \times 10^{-4}$	64	$2{\cdot}5 \times 10^{-2}$	62	$8{\cdot}4 \times 10^{-5}$	69
(15, 11) (15, 1)	0	69	0	62	1×10^{-6}	57	$7{\cdot}6 \times 10^{-5}$	66
(15, 10)	$1{\cdot}9 \times 10^{-5}$	66	5×10^{-4}	66	$2{\cdot}8 \times 10^{-2}$	66	$4{\cdot}9 \times 10^{-5}$	63
(15, 7)	$8{\cdot}8 \times 10^{-6}$	64	1×10^{-4}	41	$2{\cdot}7 \times 10^{-2}$	39	$3{\cdot}5 \times 10^{-6}$	44
(15, 14) (15, 1)	$5{\cdot}9 \times 10^{-4}$	89	$7{\cdot}7 \times 10^{-4}$	82	$8{\cdot}2 \times 10^{-4}$	76	$1{\cdot}5 \times 10^{-3}$	89
(15, 11) (15, 1)	0	70	$5{\cdot}3 \times 10^{-3}$	64	$1{\cdot}2 \times 10^{-6}$	59	$7{\cdot}8 \times 10^{-5}$	70
8 code	$4{\cdot}9 \times 10^{-4}$	89	$5{\cdot}3 \times 10^{-4}$	81	$6{\cdot}4 \times 10^{-4}$	74	$1{\cdot}5 \times 10^{-3}$	90

S = Sink-bit-error rate
E = Transmission efficiency

Upper section: 4-block delay before code change
Lower section: 1-block delay before code change

Table 7
PERFORMANCE OF AUTOMATIC 8-CODE VARIABLE-REDUNDANCY SYSTEM, WITH SOME COMPUTED FIXED-REDUNDANCY-CODE PERFORMANCES FOR COMPARISON

Test	Bit-error-rate	Block-error-rate ($n = 15$)	8-code variable redundancy		(15, 14) $E = 93\%$	(15, 11) $E = 73\%$	(15, 7) $E = 47\%$
			U	*E* %	*U*	*U*	*U*
28	$3{\cdot}1 \times 10^{-3}$	$4{\cdot}6 \times 10^{-2}$	0	34	1×10^{-3}	0	0
29	$1{\cdot}8 \times 10^{-3}$	$1{\cdot}4 \times 10^{-2}$	$2{\cdot}2 \times 10^{-3}$	91	8×10^{-3}	$3{\cdot}9 \times 10^{-3}$	$3{\cdot}8 \times 10^{-4}$
30	$5{\cdot}9 \times 10^{-4}$	$4{\cdot}4 \times 10^{-3}$	$3{\cdot}3 \times 10^{-4}$	90	$1{\cdot}7 \times 10^{-3}$	$1{\cdot}6 \times 10^{-4}$	$1{\cdot}3 \times 10^{-5}$
31	$6{\cdot}6 \times 10^{-2}$	$4{\cdot}8 \times 10^{-1}$	$1{\cdot}3 \times 10^{-1}$	81	$1{\cdot}9 \times 10^{-1}$	$1{\cdot}9 \times 10^{-2}$	$7{\cdot}2 \times 10^{-3}$
32	3×10^{-2}	$2{\cdot}6 \times 10^{-1}$	$3{\cdot}4 \times 10^{-2}$	84	$8{\cdot}1 \times 10^{-2}$	$3{\cdot}9 \times 10^{-3}$	$2{\cdot}3 \times 10^{-3}$
33	$1{\cdot}7 \times 10^{-2}$	$1{\cdot}2 \times 10^{-1}$	$2{\cdot}4 \times 10^{-2}$	85	$3{\cdot}8 \times 10^{-2}$	$1{\cdot}9 \times 10^{-3}$	$4{\cdot}8 \times 10^{-4}$
34	$1{\cdot}3 \times 10^{-3}$	$1{\cdot}1 \times 10^{-2}$	6×10^{-5}	75	$3{\cdot}3 \times 10^{-3}$	$1{\cdot}9 \times 10^{-4}$	0
35	$2{\cdot}5 \times 10^{-4}$	$3{\cdot}2 \times 10^{-3}$	$1{\cdot}2 \times 10^{-4}$	76	$2{\cdot}7 \times 10^{-4}$	0	0
36	$3{\cdot}9 \times 10^{-3}$	$2{\cdot}4 \times 10^{-2}$	0	34	$1{\cdot}1 \times 10^{-2}$	$7{\cdot}7 \times 10^{-4}$	$8{\cdot}6 \times 10^{-5}$
37	$4{\cdot}2 \times 10^{-3}$	$4{\cdot}3 \times 10^{-2}$	0	33	$9{\cdot}9 \times 10^{-3}$	0	0
38	$8{\cdot}7 \times 10^{-4}$	$9{\cdot}2 \times 10^{-3}$	$1{\cdot}3 \times 10^{-3}$	86	$1{\cdot}7 \times 10^{-3}$	$5{\cdot}3 \times 10^{-4}$	0
39	$7{\cdot}2 \times 10^{-4}$	$2{\cdot}3 \times 10^{-3}$	$8{\cdot}5 \times 10^{-4}$	40	$6{\cdot}9 \times 10^{-3}$	$1{\cdot}9 \times 10^{-3}$	$1{\cdot}8 \times 10^{-3}$
40	$5{\cdot}1 \times 10^{-3}$	$2{\cdot}8 \times 10^{-2}$	$1{\cdot}6 \times 10^{-3}$	37	$7{\cdot}3 \times 10^{-3}$	$5{\cdot}9 \times 10^{-3}$	$3{\cdot}1 \times 10^{-3}$
41	$1{\cdot}2 \times 10^{-2}$	$6{\cdot}8 \times 10^{-2}$	5×10^{-3}	46	$2{\cdot}3 \times 10^{-2}$	$6{\cdot}8 \times 10^{-3}$	$4{\cdot}2 \times 10^{-3}$
42	$7{\cdot}8 \times 10^{-3}$	$7{\cdot}0 \times 10^{-2}$	$1{\cdot}7 \times 10^{-3}$	49	$1{\cdot}3 \times 10^{-2}$	7×10^{-3}	0
43	$4{\cdot}6 \times 10^{-3}$	$2{\cdot}8 \times 10^{-2}$	$3{\cdot}4 \times 10^{-3}$	34	$1{\cdot}6 \times 10^{-2}$	0	0
44	$8{\cdot}3 \times 10^{-3}$	$5{\cdot}7 \times 10^{-2}$	3×10^{-3}	26	$1{\cdot}9 \times 10^{-2}$	$3{\cdot}6 \times 10^{-3}$	$1{\cdot}4 \times 10^{-3}$
45	$1{\cdot}1 \times 10^{-2}$	$5{\cdot}3 \times 10^{-2}$	$1{\cdot}2 \times 10^{-3}$	37	$2{\cdot}1 \times 10^{-2}$	$2{\cdot}5 \times 10^{-3}$	9×10^{-4}
46	$3{\cdot}1 \times 10^{-2}$	$1{\cdot}5 \times 10^{-1}$	$1{\cdot}5 \times 10^{-2}$	39	6×10^{-2}	$1{\cdot}1 \times 10^{-2}$	$7{\cdot}5 \times 10^{-3}$

U = Sink-block-error rate
E = Transmission efficiency

errors. Selective fading had a more serious effect on the error rate than did flat fading.

The computed results (Tables 4 and 5) show that fixed-redundancy coding improves the error rate, on average by a factor of about 100. With the short block lengths used, however, the efficiency is poor for the more powerful codes. Simulations of variable-redundancy schemes show a distinct improvement in efficiency, with no loss of error control (Table 6). Interleaving offers some degree of improvement, but is only worthwhile for small degrees of interleave.

The improvement due to the use of variable redundancy is confirmed by the results for the experimental automatic variable-redundancy system (Tables 7 and 8). The results would have been even better if:

(*a*) burst-control codes had been used, instead of random-error-control codes
(*b*) codes less sensitive to synchronisation-slip (framing) errors had been used.[4]

It is also clear (Table 8) that simple 2-code variable-redundancy-coding schemes offer advantages of realisation and control, as compared with the more sophisticated system, without appreciable loss of error control, and with some increase in efficiency of transmission. It may not even be necessary to

Table 8
PERFORMANCE OF 2-CODE AND 8-CODE VARIABLE-REDUNDANCY SYSTEMS WITH A.R.Q.

Test	Fixed redundancy (15, 14)		2-code (15, 14) (15, 1)		2-code (15, 11) (15, 1)		8-code	
	S	E	S	E	S	E	S	E
		%		%		%		%
28	$1{\cdot}2 \times 10^{-4}$	88	$1{\cdot}1 \times 10^{-4}$	85	0	66	$1{\cdot}1 \times 10^{-4}$	85
29	$1{\cdot}3 \times 10^{-4}$	92	$2{\cdot}7 \times 10^{-5}$	91	$4{\cdot}1 \times 10^{-4}$	72	$1{\cdot}2 \times 10^{-4}$	91
30	$1{\cdot}6 \times 10^{-4}$	93	$9{\cdot}8 \times 10^{-5}$	92	0	72	$8{\cdot}7 \times 10^{-5}$	92
31	$2{\cdot}4 \times 10^{-2}$	44	$5{\cdot}8 \times 10^{-3}$	21	0	14	$1{\cdot}5 \times 10^{-3}$	14
32	$6{\cdot}8 \times 10^{-3}$	61	$9{\cdot}6 \times 10^{-4}$	48	0	38	$7{\cdot}1 \times 10^{-4}$	47
33	$3{\cdot}4 \times 10^{-3}$	77	$1{\cdot}7 \times 10^{-3}$	68	$1{\cdot}2 \times 10^{-4}$	53	$1{\cdot}7 \times 10^{-3}$	68
34	$2{\cdot}3 \times 10^{-3}$	92	$1{\cdot}7 \times 10^{-5}$	91	$2{\cdot}9 \times 10^{-5}$	71	$9{\cdot}7 \times 10^{-6}$	91
35	$1{\cdot}7 \times 10^{-5}$	92	$1{\cdot}7 \times 10^{-5}$	92	0	72	$1{\cdot}7 \times 10^{-5}$	92
36	$1{\cdot}6 \times 10^{-3}$	90	$2{\cdot}3 \times 10^{-4}$	87	$2{\cdot}6 \times 10^{-5}$	68	$1{\cdot}5 \times 10^{-5}$	87
37	$5{\cdot}7 \times 10^{-4}$	86	$1{\cdot}6 \times 10^{-4}$	82	0	65	0	82

S = Sink-bit-error rate
E = Transmission efficiency

monitor the a.r.q. (block-error) rate; a simple criterion like 'if two successive blocks with errors occur, change to the more powerful code', and vice-versa, could be used. Again, the computed results (Table 5), confirm this.

The h.f. medium is one of the most severe channels for high-speed data transmission, but the results of this study show that simple, reliable and efficient high-speed data communication can be realised by using a set of short, fixed-length block-error-detection codes, with a.r.q. for error correction, and with a variable-redundancy scheme operating via the a.r.q.-feedback link. The method can also be applied to other types of channel.

Several ways of overcoming some of the disadvantages of variable-redundancy coding are being investigated. For example, the problem of the variable input data rate might be solved by using a multiplex system, with the channels assigned different degrees of error control, depending on the volume and priority of the traffic, as well as on the channel conditions. A deeper study of codes for variable redundancy is also under way involving an investigation of:

(*a*) code sets with distinct code words
(*b*) comma-free variable-redundancy codes
(*c*) variable-redundancy convolutional codes

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