Soft-decision error-correction coding schemes for HF data transmission.

R.M.F. Goodman, A.D. Green, and A.F.T. Winfield

Summary

This paper is concerned with the use of soft-decision decoding of error-correcting codes in the context of HF data transmission. The use of soft-decision information from the data modem results in an improvement in the performance of a forward-error-correction scheme, when compared with hard-decision decoding, without any further redundancy penalty. In the paper, we estimate the theoretical improvements that can be expected from soft-decision decoding of block and convolutional codes, in terms of both random and burst error-correcting power. Also, this is related to expected coding gains for the Gaussian and Rayleigh fading channels. In addition, the performance of several low-complexity soft-decision coding schemes are investigated. Computer simulation results, using real error data recorded from a Kineplex type modem, are presented and discussed.

Dr. R.M.F. Goodman, Mr. A.D. Green, and Mr. A.F.T. Winfield are with the Department of Electronic Engineering, University of Hull.

1. Introduction

In a hard-decision error-control-coded binary data transmission system the receiver/demodulator makes a hard 0/1 decision on each incoming data signal before feeding the demodulated bit to the error-correction decoder. Similarly, in a multi-phase modulation system a 'hard' decision is made at each phase boundary. This procedure results in a degradation of the channel decoder's performance. A soft-decision demodulator on the other hand, assigns a 'confidence' value to each output bit, in addition to the 'hard' binary 0 or 1 decision. In essence this means that each demodulated bit is quantised into $Q > 2$ levels, rather than $Q = 2$ levels as in the hard-decision case. This confidence information can then be used to improve the error-correction decoder's performance (in terms of lower output bit error rate) without incurring any further redundancy penalty.

The use of soft-decision decoding is therefore particularly pertinent to the case of HF data transmission. This is because, due to the high channel error rates and non-Gaussian error statistics that exist on most HF data links, hard-decision decoding schemes simply do not have sufficient error-correction power per bit to provide useful coding gain. Soft-decision decoding schemes, however, can increase the correction power per bit, but at the expense of further complexity. The increase in coding gain (over that achievable with hard-decision) that can be expected by using soft-decision decoding depends on a number of factors. These include the number and spacing of the quantisation levels, the decoding algorithms used, and the channel characteristics. It will be shown later, however, that soft-decision decoding can (at the most) double the correction power per bit of a code, and therefore achieve a performance that tends to the optimum maximum-likelihood decoder. This increase in power is certainly worth having, particularly in the case of HF data transmission.

The main objection to the use of soft-decision decoding is one of hardware complexity. This is because, in addition to the decoder having to handle and store J bits (where 2^J=Q) instead of 1 bit per decision, existing soft-decision algorithms are much more complex than hard-decision algorithms. This is particularly true in the case of block codes, as opposed to convolutional codes where the Viterbi algorithm provides an effective (although still severely complexity limited) soft-decision decoding scheme. In this paper we investigate the performance of several block and convolutional soft-decision decoding schemes, with the accent on low decoder complexity.

This paper develops in the following way. Firstly, we estimate the performance improvements that can be expected from soft-decision decoding of block and convolutional codes in terms of both random and burst error-correcting power. Also, this is related to expected coding gains for the Gaussian and Rayleigh fading channels. The characteristics of the HF channel which have major influences on the choice of any soft-decision error-control scheme are then discussed. Next, several practical soft-decision decoding algorithms, some of which stem from our previous work, are described. Finally, the performance of the low-complexity schemes described are assessed for an HF channel, using error data recorded from a Kineplex type modem.

2. Expected improvements in performance due to soft-decision decoding

Given a block or convolutional error-control scheme, let us assume that the soft-decision demodulator quantises each output digit v_i to 2^J=Q levels, symmetrically spaced about the hard 0/1 decision boundary. The estimate of the i^{th} received binary digit is given by the soft-decision J-bit byte:

$[v_i] = [v_i^1 v_i^2 \ldots v_i^J]$, where the square brackets indicate a soft-decision quantity. The first bit of $[v_i]$ is the hard-decision estimate, and the remaining J-1 bits give an indication of the confidence of that estimate. The confidence of the hard-decision may be defined as the J-1 bit byte: $[c_i] = [c_i^1 c_i^2 \ldots c_i^{J-1}]$, where $[c_i] = [v_i^2 v_i^3 \ldots v_i^J]$ if $v_i^1 = 1$, or $[c_i] = [v_i^2 v_i^3 \ldots v_i^J] \oplus [11 \ldots 1]$ if $v_i^1 = 0$. Thus the confidence of a particular received bit can vary from $[c_i] = [00 \ldots 0]$ (least confident, nearest to the hard-decision 0/1 boundary), to $[c_i] = [11 \ldots 1]$ (furthest away from the boundary). Alternatively, we may consider that the demodulator output soft-decision digit $[v_i]$ gives an estimate of the soft-decision error digit $[e_i]$ which has been added to the transmitted digit $[u_i]$. Hence, $[v_i] = [u_i] \oplus [e_i]$, where $[u_i] = [00 \ldots 0]$ or $[11 \ldots 1]$ only. The value of the soft-decision error digit in <u>levels</u> can therefore lie between 0 and (Q-1), and a value of $\geq$ (Q/2) constitutes an 'error' in the hard decision sense.

We may now form an estimate of the improvement in random-error correcting power when soft-decision decoding is used. Consider a block or convolutional code whose decoding constraint length is n bits. If the hard minimum distance of the code is d_h over n bits, then its bounded-distance hard correcting power is the largest integer $t_h \leq \{(d_h-1)/2\}$. This gives a per bit hard correction power of t_h/n. In the soft-decision sense code words (paths) are $\geq d_s = (Q-1)d_h$ soft-decision <u>levels</u> apart, and therefore the bounded distance guaranteed soft-decision error-correction power in <u>levels</u> is $t_s \leq \{d_s - 1)/2\}$. The smallest number of levels that constitutes an error in the hard decision sense is Q/2, and the maximum number of 'hard' errors that can be corrected is therefore $t_s/(Q/2) = \frac{2}{Q}\{(d_s - 1)/2\} = \frac{1}{Q}\{(Q - 1)d_h - 1\} \simeq d_h$ for Q large. Thus, the per bit correction power has approximately doubled from t_h/n to d_h/n.

It should be noted that the doubling in correction power is an upper bound on the improvement due to soft-decision, and will only be achieved at very high signal-to-noise ratios. In general, at low signal-to-noise ratios the average improvement will be significantly less than this.

2.1 Improvement on the Gaussian channel

Consider the Gaussian channel. If the (single-sided) noise power density is given by N_o, the signal-to-noise ratio is given by $\gamma = E/N_o$, and the bit probability of error is given by the $\mathcal{Q}$ function:

$$p = \int_{\sqrt{2\gamma_b R}}^{\infty} [\exp(-x^2/2)/\sqrt{2\pi}]dx \triangleq \mathcal{Q}(\sqrt{2\gamma_b R}) \tag{1}$$

where $\gamma_b = E_b/N_o = \gamma/R$ is the normalised signal-to-noise ratio per information bit, and R is the inverse of the bandwidth expansion factor (that is, the code rate). The probability of bit error for a hard-decision coded system can be lower bounded by

$$P_e > w_{d_h} p^{d_h}(1 - p)^{n - d_h} \tag{2}$$

where d_h is the minimum distance of the code over the decoding constraint length n, and w_d is the number of bit errors contributed by incorrect decoding of a code path of distance d_h.

Asymptotically, at high signal-to-noise ratio,

$$Q(\sqrt{2\gamma_b R}) \simeq \exp(-\gamma_b R) \tag{3}$$

and hence equation (2) reduces to

$$P_e \simeq w_d \exp(-\gamma_b R d_h) \tag{4}$$

Assuming that soft-decision decoding effectively doubles the distance of a code, that is, $d_s = 2d_h$, then for

$$P_{e(soft)} = P_{e(hard)}, \text{ we require}$$

$$\gamma_{b(soft)}\ R2d_h = \gamma_{b(hard)}\ Rd_h, \text{ that is,}$$

$$\frac{\gamma_{b(hard)}}{\gamma_{b(soft)}} = 2$$

which indicates a 3dB improvement in coding gain. This is similar to the improvement obtained in changing a d_h-th order diversity system to a $2d_h$-th order diversity system[1].

Also, P_e uncoded $\simeq \exp(-\gamma_b)$ from (3) which shows that the upper limit on coding gain is given by:

$G_c < 10\log Rd_h$ (dB) for hard-decision decoding and

$G_c < 10\log 2Rd_h$ (dB) for soft-decision decoding.

At the opposite extreme, that is, for the very noisy channel, it has been shown[2] that a performance loss of about 2dB is incurred when hard-decision decoding is used as opposed to infinitely quantised soft-decision decoding. Also, the degradation involved in using the much more practical equal-spacing 8-level quantisation is only about 0.2dB[3].

Thus, at high error rates on a Gaussian channel we expect a maximum soft-decision coding gain of about 1.8dB. At low error-rates a 2dB improvement in signal-to-noise ratio corresponds to a reduction in output bit error rate of approximately 2 orders of magnitude for uncoded binary antipodal signalling on the Gaussian channel. At high error rates, however, the uncoded performance curve flattens out, and a characteristic of coded transmission is that at some value of E_b/N_o a coded transmission will perform worse than an uncoded one. The coded performance curve effectively 'crosses over' the uncoded curve. In this high error rate region, the uncoded output bit error rate is only improved by about a factor of 3 for a 2dB improvement in E_b/N_o. Thus, although we expect an improvement in performance due to soft-decision decoding at high error rates, this improvement will not be large.

2.2 The Rayleigh Channel

The bit error probability for the coherent Rayleigh fading channel[1] is given by:

$p = \frac{1}{2}(1 - \mu)$ where

$$\mu = \left(\frac{\gamma_b R}{\gamma_b R+1}\right)^{\frac{1}{2}}$$

The lower bound on bit error probability (equation 2) is then given by

$$P_e > w_{d_h} \left(\frac{1-\mu}{2}\right)^{d_h} \left(\frac{1+\mu}{2}\right)^{n-d_h} \qquad (5)$$

which for high signal-to-noise ratios becomes:

$$P_e \approx w_{d_h} \left(\frac{1}{4\gamma_b R}\right)^{d_h} \approx K\left(\frac{1}{\gamma_b}\right)^{d_h} .$$

Assuming $d_s = 2d_h$, then for equal output bit error rate we have

$$\gamma_{b(hard)} = \gamma^2_{b(soft)}$$

which shows that the soft coding gain is an increasing function of E_b/N_o, and that soft-decision decoding requires approximately half the signal-to-noise ratio (in dB) to achieve the same output bit error rate as hard-decision decoding.

Soft-decision decoding on the Rayleigh fading channel is therefore theoretically capable of providing much larger soft-coding gains than in the case of the Gaussian channel. It must be noted again, however, that the expected halving in power requirement will not be achieved at low signal-to-noise ratios.

2.3 Burst Channels

It is not possible to derive a theoretical soft-decision improvement figure for a complicated time-varying channel such as the HF channel. In general, the HF channel can be considered[4] to be a diffuse-burst channel in which error bursts of medium to high density are separated by relatively short gaps with a low density of errors. As such, any coding scheme that is used on the HF channel must have both burst-and-random error-correction capability. We have already shown that the random error-correction power of a code is improved by the use of soft-decision decoding. It is therefore appropriate to assess the improvement in burst correcting power.

Consider a random error-correcting code with a correction power of t_h over a decoding constraint length of n bits. This implies that all bursts of length $b \leq t_h$ or less can be corrected. If we assume that, asymptotically, soft-decision decoding doubles the power of the code, then the code will now be able to correct any combination of two or less bursts of length $b \leq t_h$, or at single burst of length $2t_h$.

Interleaving is a powerful technique which can be used to provide both burst-and-random correction power, whilst still using a random error-correction decoder. If the above basic code is interleaved to a depth of λ then the hard-decision power of the interleaved code over $n\lambda$ bits is such that any combination of t_h or fewer bursts of length λ or less, can be corrected. Thus the application of soft-decision should allow (at a maximum) any combination of $2t_h$ or fewer bursts of length λ or less to be corrected. It is therefore the multiple-burst correcting power within a given constraint length that is significantly increased by soft-decision decoding rather than the single burst correcting power.

A well known[5] bound on burst correcting capability (b) relative to error-free guard space (g), which holds for both block and convolutional codes is given by:

$$\frac{g}{b} \geq \frac{1 + R}{1 - R} = 3 \text{ for a } \tfrac{1}{2} \text{ rate code.}$$

In general, an interleaved random-error-correcting code does not approach this bound closely. For example, the (23,12) perfect Golay code has $b = 3$ and $g = 20$ giving $g/b = 6.7$. However, if the use of soft-decision increases the burst capability by only one to $b = 4$ on average, then $g/b = 4.75$, a significant improvement.

If, in the limit, we assume that soft-decision decoding can double the single burst correcting power of a code then

$$(g/b)_{soft} \geq \tfrac{1}{2}\left(\frac{1 + R}{1 - R}\right) .$$

At high error rates the improvement due to soft decision will not be mainly in the single-burst correction capability but rather in the multiple-burst correction capability. This implies that soft-decision will show the most improvement on a diffuse burst channel rather than a dense burst/long gaps channel.

3. HF Channel Characteristics

The error characteristics experienced in HF data transmission depend not only on the channel characteristics at a particular time but also on the type of modem used. In this paper we consider a Kineplex type modem operating at 2400 bits per second. The data is transmitted in 48 bit parallel blocks or frames, using orthogonal multi-subcarrier phase shift keying. Soft-decision information on each demodulated bit is available from the modem.

There are several characteristic types of error events due to this modem structure.

(i) Random errors.

(ii) Errors which occur in the same place in repeated frames due to stationary frequency selective fading on one or more sub-carriers within the band, thus causing isolated repetitive bursts.

(iii) Sweeping frequency selective fades which traverse the band causing errors in repeated frames but in different frame positions.

(iv) Flat fades across the band which cause large bursts of errors.

Figure 1 shows a particular received sequence of bits, an error being indicated by an asterisk. It can be seen that one particular sub-carrier within the frame is experiencing a bad stationary selective fade, and contributing errors in almost every successive frame.

Figure 3 shows a received sequence of bits in which two selective fades are sweeping across the band, causing errors in successive frames but at different frequencies.

Figure 2 shows a bad flat and selective fading situation which is causing frequent bursts of errors.

In addition, all the figures show a varying amount of residual errors.

As a consequence of these error characteristics it can be seen that any coding scheme used on a bit-by-bit basis must have both burst and random error correction power, and that if a random error-correcting code is used, it must be interleaved in both time and frequency.

4. Soft-decision decoding schemes

Given a block or convolutional code operating over a decoding constraint length of n bits, the optimum method of decoding is maximum-likelihood decoding, which for the binary symmetric channel is equivalent to minimum distance decoding. A minimum distance hard-decision decoder therefore attempts to find the codeword (path), u, nearest in terms of Hamming distance to the received sequence v. That is, the code sequence which satisfies

$$\min \{ \sum_{i=1}^{n} (v_i \oplus u_i) \} = \min \{ \sum_{i=1}^{n} e_i \} .$$

Alternatively, this is equivalent to finding the minimum weight error pattern e which will turn the received sequence v into a valid code sequence u.

Similarly, we may define an optimum soft-decision minimum distance decoder (which approximates to a maximum likelihood decoder) as one that attempts to find the code sequence at minimum soft-distance (minimum number of level errors) from the received sequence v. That is,

$$\min \{ \sum_{i=1}^{n} ([v_i] \oplus [u_i]) \} = \min \{ \sum_{i=1}^{n} [e_i] \}$$

We now briefly describe several soft-decision decoding schemes which approximate to this optimum behaviour; but have low complexity. The performance of these codes on the HF channel is assessed in the next section. Each code is identified by an abbreviation.

4.1 Soft-decision threshold decoding

Recently, we have proposed a soft-decision version of the well known hard-decision majority decision threshold decoding algorithm[6] which is suitable for both block and convolutional codes[7,8]. The convolutional codes investigated in this class are:

(i) A half-rate random error-correcting code with $n = 14$, $t_h = 2$, and generator sequence g = 11000100000101. (T14H)

(ii) A half-rate random error-correcting code with $n = 24$, $t_h = 3$, and generator sequence g = 110000000000010100010101. (T24H)

(iii) A half-rate burst and random-error-correcting diffuse code[9] with $n = 6\beta+2$, which can correct any 2 random errors within n consecutive bits, or any burst of length $\leq 2\beta$ relative to a guard space of n bits. This code has $g = 11(00)_{\beta-1}$ $01(00)_{\beta-1}\ 01(00)_{\beta}01$. (TDIFH)

(iv) A one-third rate code with $n = 15$, $t_h = 3$, and g = 111 010 001 001 001. (T15T).

4.2 The (23,12) perfect Golay code

The perfect Golay code is a triple-error-correcting code with 12 information digits in the decoding constraint length of 23. We have developed a soft-decision minimum distance decoding algorithm for this code which is based on error-trapping decoding[10]. The algorithm uses permutation decoding[11] in a predictive manner such that both burst and random error-correction is possible. (GOLAY)

4.3 Sub-optimum soft-decision minimum-distance decoding of convolutional codes

Optimum minimum distance soft-decision decoding of convolutional codes can be achieved by means of the Viterbi algorithm, provided that the encoding constraint length is limited to about 14 bits so that decoder complexity does not become excessive. Viterbi decoders have been investigated by several researchers, and these have been shown to exhibit reasonably good performance over both satellite and HF channels[12,13].

Recently, however, we have proposed a hard-decision minimum distance decoding algorithm, that is efficient for both short and long codes[14]. The algorithm consists of two main processes. A direct mapping scheme which can locate the minimum distance path without any searching, and an efficient path searching scheme. In this paper we investigate the performance of two very low complexity but sub-optimum forms of the algorithm.

(i) Hard-decision direct mapping decoding[15,16].

(ii) Soft-decision efficient path searching[17,18].

Both decoding methods are applied to the following codes:

(i) A half-rate code with $n = 22$, $t_h = 3$, and g = 1101000100010001010000 (MD22H)

(ii) A one-third rate code with $n = 21$, $t_h = 4$, and g = 111001010010001011011 (MD21T)

4.4 Interleaving

In order to combat the effects of flat and selective fading in a multi-subcarrier HF transmission system, it is necessary to interleave the transmitted bits in both time and frequency. In practise this involves reading the encoded data stream into a 48 by Δ array, where Δ is the interleaving depth in frames; and reading out the bit stream to the modem in a diagonal manner. The inverse operation of de-interleaving is performed at the receiver, before feeding bits to the error-correction decoder. Given a fixed parallel frame of 48 bits, there exists a trade-off between the time and frequency separation of adjacent bits in the de-interleaved stream. For example, if $\Delta = 48$ then bits are separated in time by 48 frames, but will have been transmitted on the same sub-carrier. Thus, stationary selective fades will cause long bursts of errors in the de-interleaved bit stream. In this paper we consider interleaving to a depth of 8 frames, which gives adjacent bits a separation of 8 bits in time, and $\Delta/8 = 6$ bits in frequency across the frame.

5. Performance Results

In this section we investigate the performance of the coding schemes described in the last section. The decoding schemes were computer simulated using runs of data-independent soft-decision error sequences, recorded from the modem operating over a real HF link. Results are presented for three runs which display characteristic error conditions, as summarised in Table 1.

Table 1.

	Frames	Bits	Errors	Error rate	length (secs)	characteristics
RUN 1	440	21120	964	1 in 22	8.8	stationary selective fading
RUN 2	440	21120	1951	1 in 11	8.8	Flat fades
RUN 3	440	21120	1350	1 in 16	8.8	Sweeping selective fades

Figures 4, 5 and 6 are histograms of the error sequences for the runs. Each horizontal division corresponds to two frames or 96 bits. Each asterisk (plotted vertically) corresponds to a hard-decision error.

Each coding scheme described in section 4 was tried on each of the three runs, using both hard and soft decision decoding, and both interleaved and non-interleaved operation. The results of these simulations are plotted in figures 7, 8 and 9, in histogram form, corresponding to runs 1, 2 and 3 respectively. The vertical axis corresponds to the normalised relative output bit error rate for each coding scheme. That is:

$$\left(\frac{1}{F}\right) = \frac{\text{decoder output data errors}}{\text{channel errors} \times \text{Code Rate}}$$

where F can be considered to be the improvement factor in output bit error rate due to the use of coding. Both hard-decision and soft-decision results are presented, the unshaded area indicating the soft-decision result and the unshaded plus shaded area indicating the hard-decision result. The shaded area therefore indicates the relative improvement offered by soft-decision over hard-decision decoding. The abbreviations used to identify the coding schemes are as indicated in Section 4.

In addition, figure 10 shows results averaged over all three runs.

6. Discussion

From the results presented, it can be seen that soft-decision decoding yields useful performance gains for all the coding schemes tried. Averaged over all results the output bit error rate is improved by a factor of about 2½ by the use of soft-decision decoding. This result is in excellent agreement with the expected improvement given in section 2.1, for high error rates.

Comparing the different coding schemes, it can be seen that the more nearly optimum decoding schemes perform best, as expected. For example, with the threshold decoders, the n = 14 half-rate scheme and the n = 15 one-third rate scheme both perform reasonably well. This is because threshold decoding performs nearly as well as full minimum distance decoding for short constraint length, low power codes. As constraint lengths are increased, threshold decoding wastes much of the power of a code, resulting in inferior performance at low error rates. This is exemplified by the n = 24 half-rate threshold decodeable code. Also, the diffuse decoding scheme, which is essentially low power, performs as well as the interleaved n = 14. This is to be expected as both schemes have $t_h = 2$.

In general, as constraint lengths are increased so is correction power. However, this is only true if the decoding scheme utilises the full minimum distance and hence correction power of the code. This can be seen by the way in which the Golay scheme significantly outperforms the sub-optimum minimum distance path searching scheme. Both codes operate over roughly the same constraint length, but the Golay scheme is much closer to the optimum maximum likelihood decoder over this constraint length. In addition, at high error rates, block codes perform better than convolutional codes, because the output burst of errors due to a decoding error is restricted to one block length. A convolutional code may take several constraint lengths to recover from the decoding error, thus causing long output bursts of errors to occur.

The minimum distance soft-decision path searching scheme when used on the one-third rate code, however, performs much better than in the half-rate case. This scheme was the only one to achieve zero output errors (Run 3).

In conclusion, it can be seen that for half-rate codes used at high error rates on the HF channel, the best performance will be obtained if an optimum soft-decision minimum distance decoding scheme is available for long constraint lengths. This requirement implies that powerful, interleaved, random-error-correcting codes should be used rather then burst-correcting schemes which are sub-optimum in their random correction power. Such algorithms are very complex for block codes, but for half-rate convolutional codes the Viterbi algorithm is capable of providing good results, as shown in reference 13. Similarly, we expect that a full soft-decision minimum distance decoding algorithm based on the algorithm in reference 14, should provide equal or superior results to that of the Viterbi algorithm. In addition, such a decoding scheme would be considerably less complex than a Viterbi decoder, for one-third rate codes.

Finally, it should be noted that if very low output bit error rates are required at high channel error rates it becomes necessary to use high redundancy codes (e.g. one-third rate). However, as it is not possible to correct for rate, as is done in the case of the Gaussian channel, it is not possible to assess whether or not coding schemes are using this redundancy effectively. A true assessment would require different modem/error-correction designs to be compared on the basis of output data bit error rate, with each design operating within a fixed channel bandwidth, and at a fixed data output speed.

7. Acknowledgements

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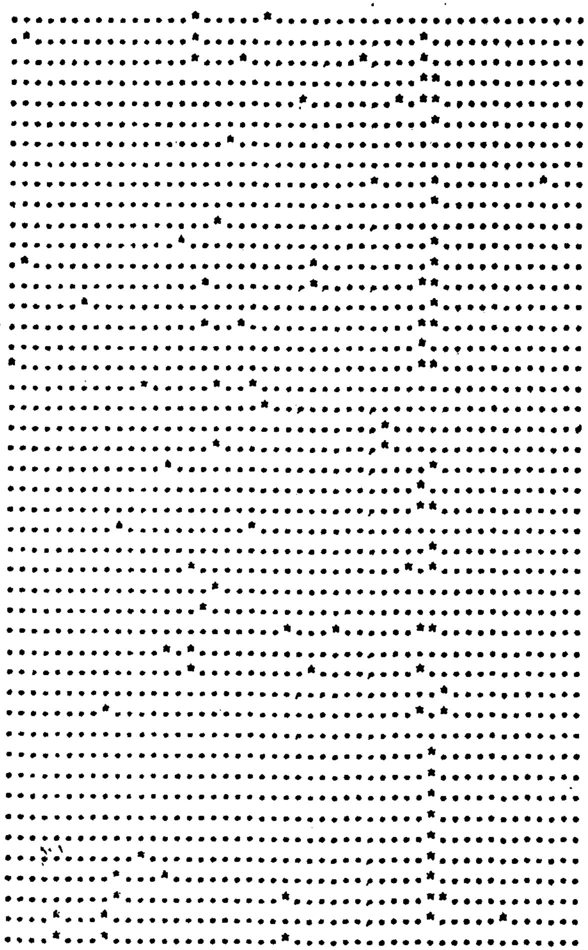


Fig. 1

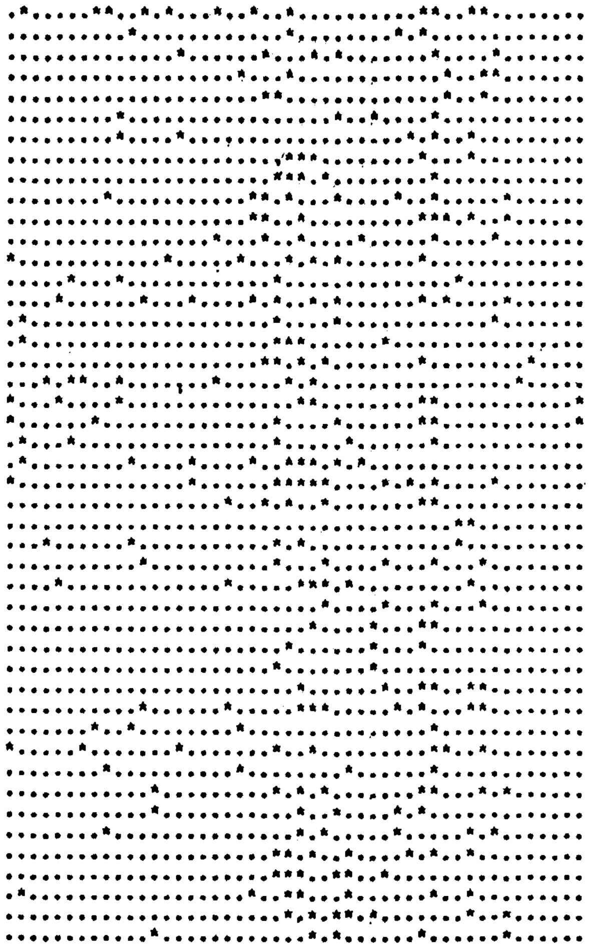


Fig. 2

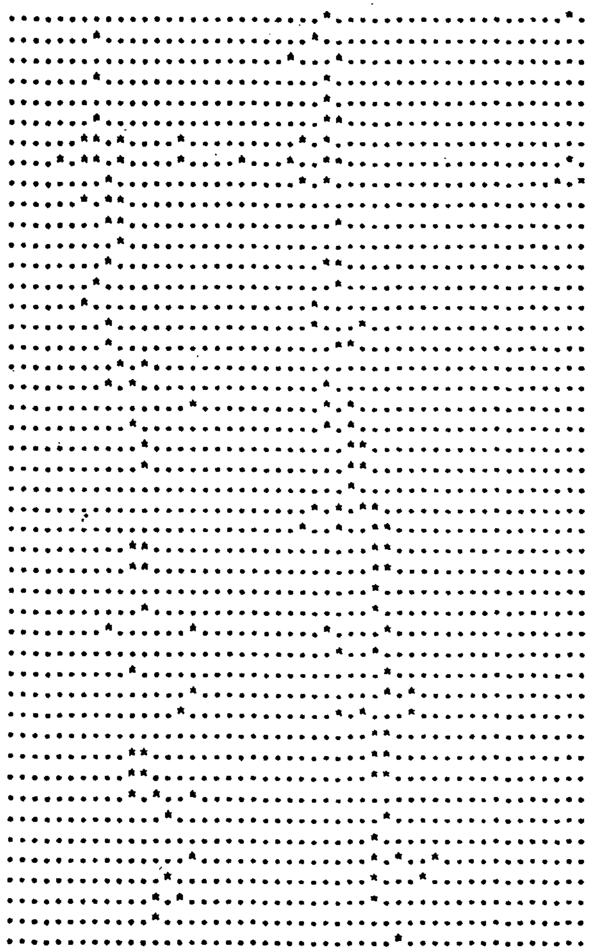
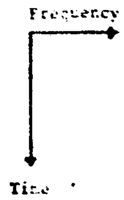


Fig. 3



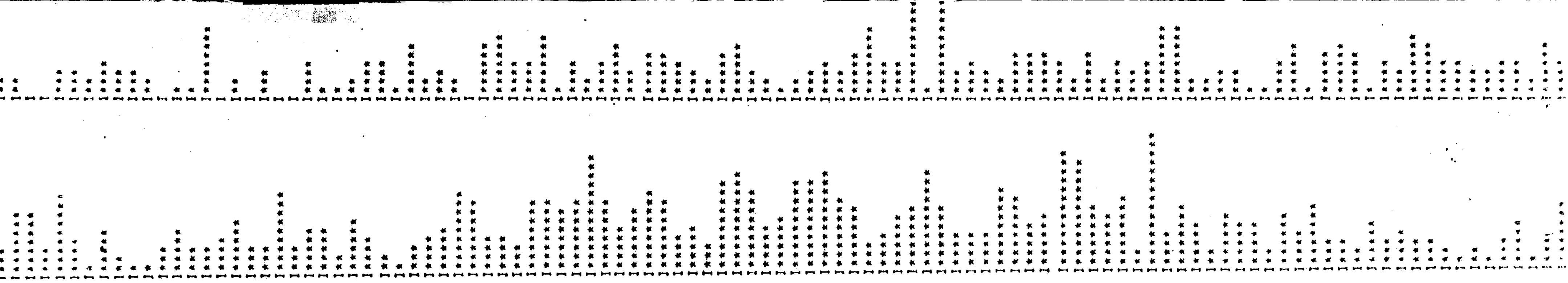


Fig.5. Run 2

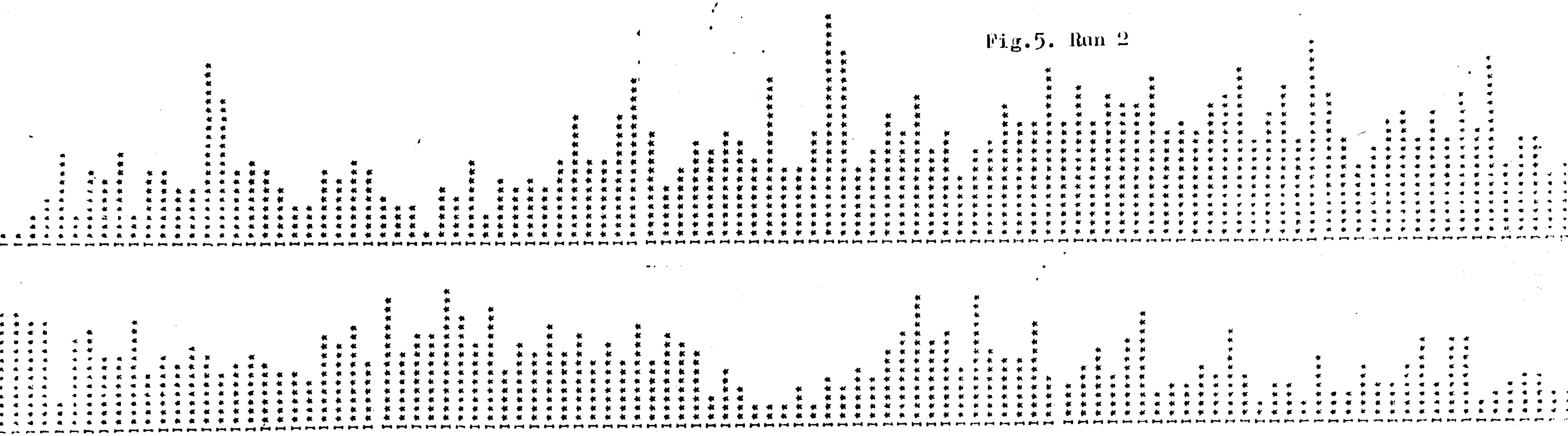


Fig.6. Run 3

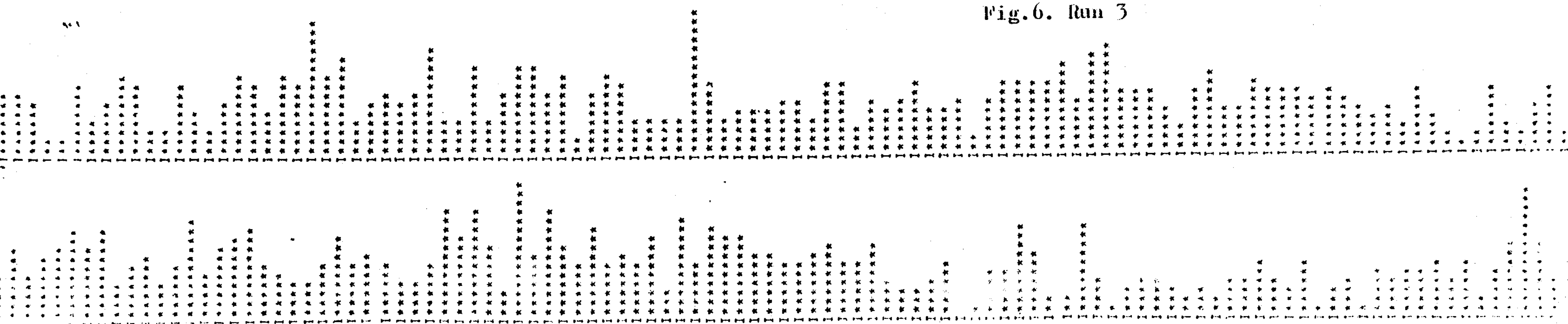


Fig.7. RUN 1

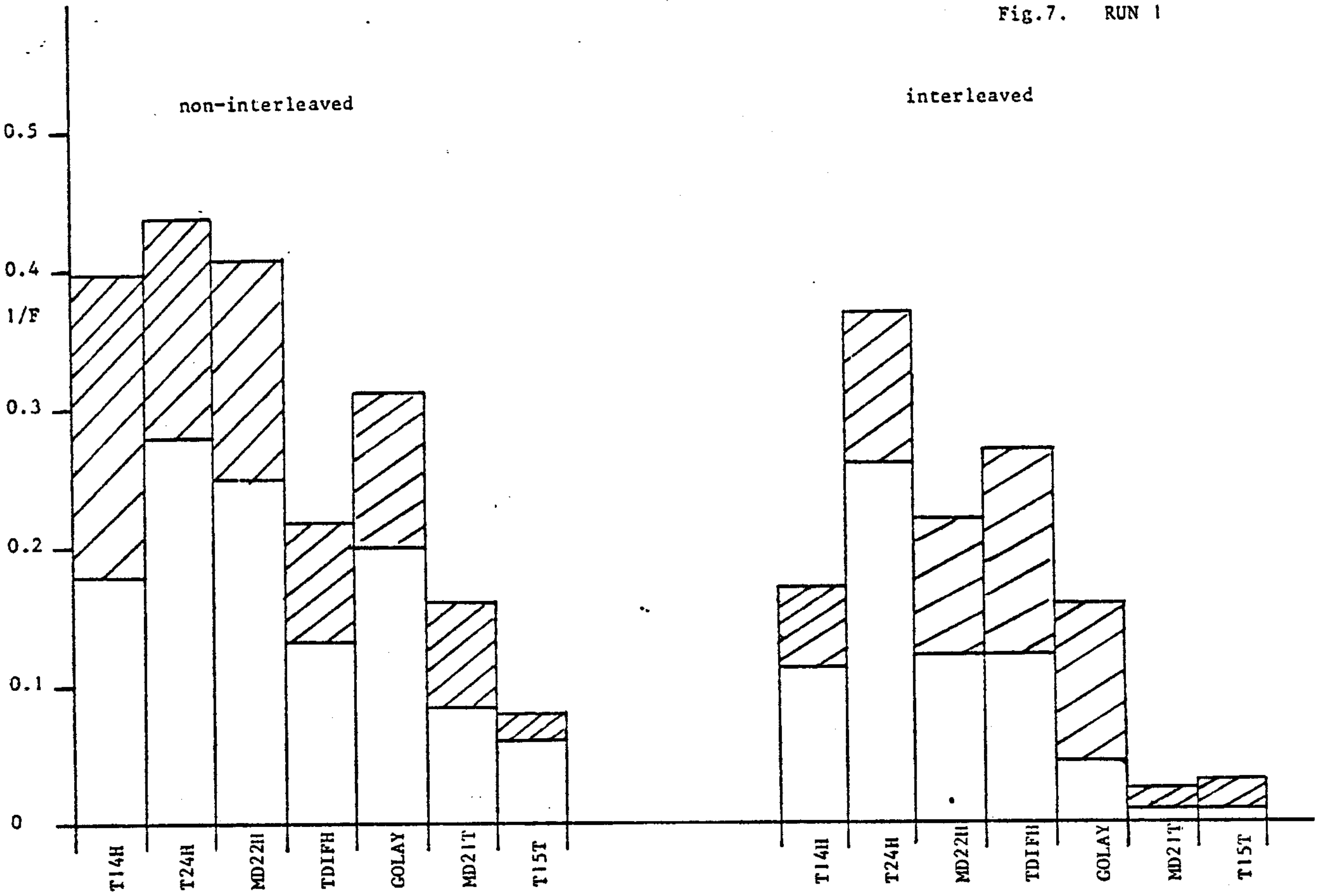
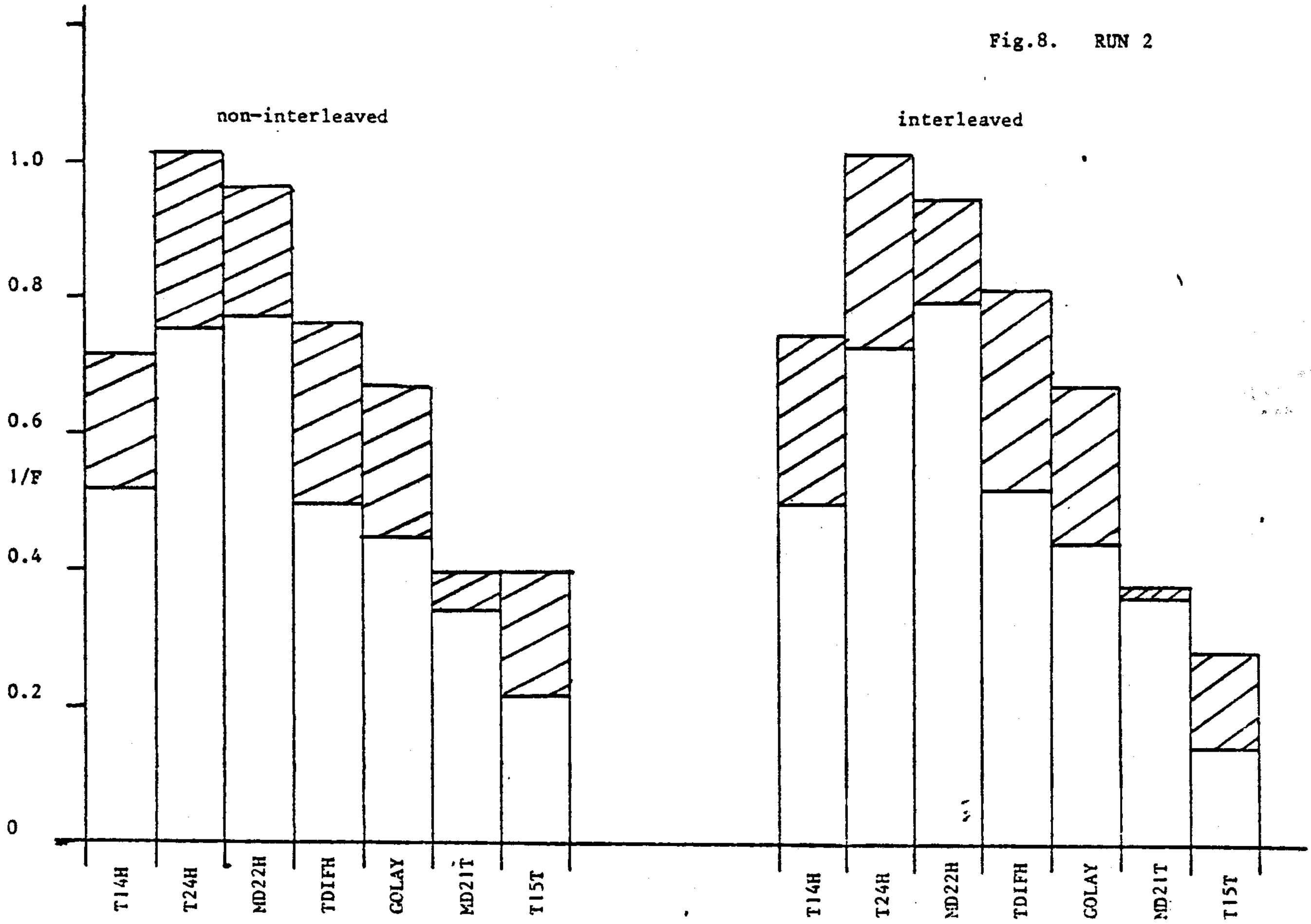


Fig.8. RUN 2



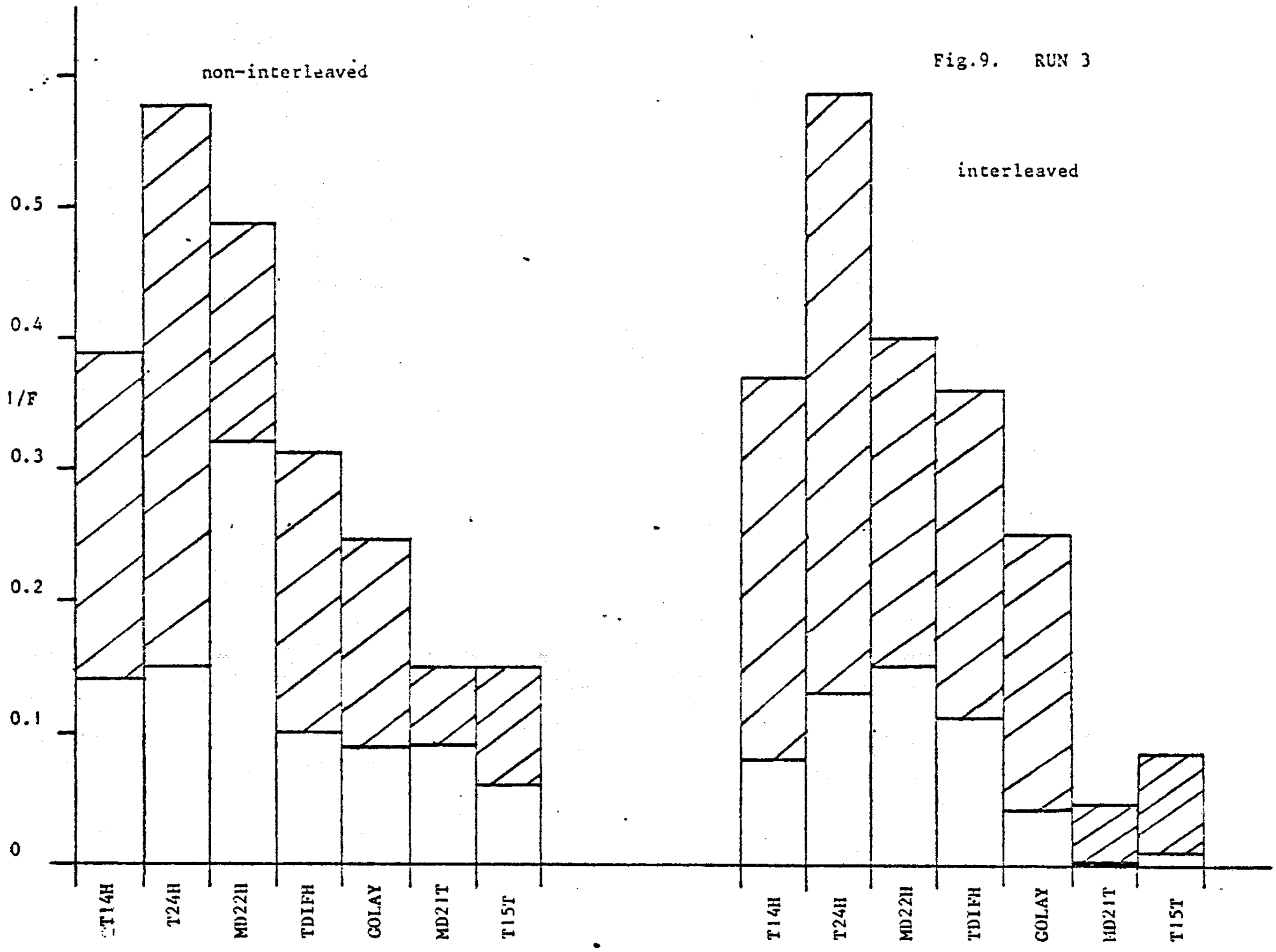


Fig.9. RUN 3

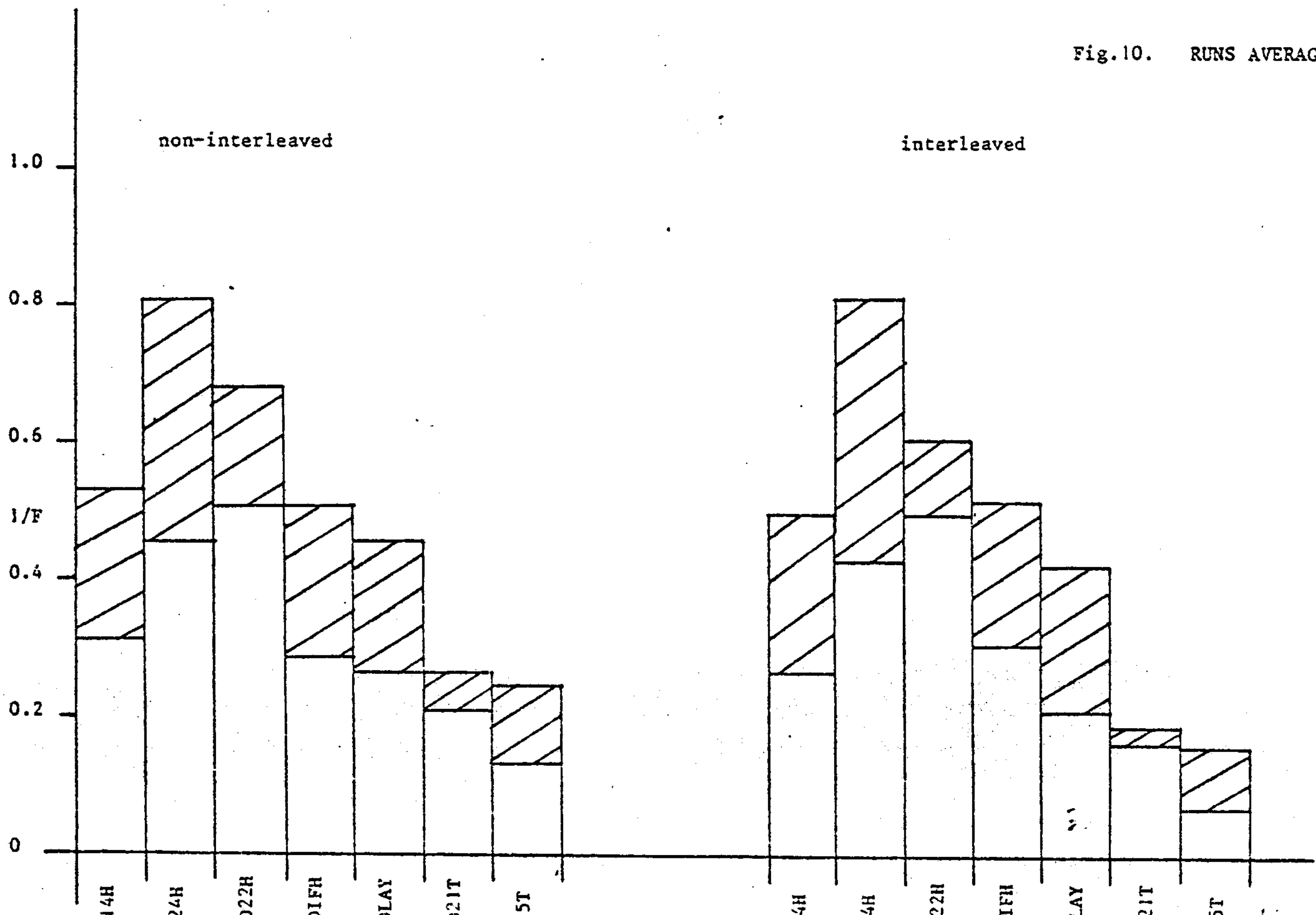


Fig.10. RUNS AVERAGED