SOFT-DECISION DECODING OF REED-SOLOMON CODES

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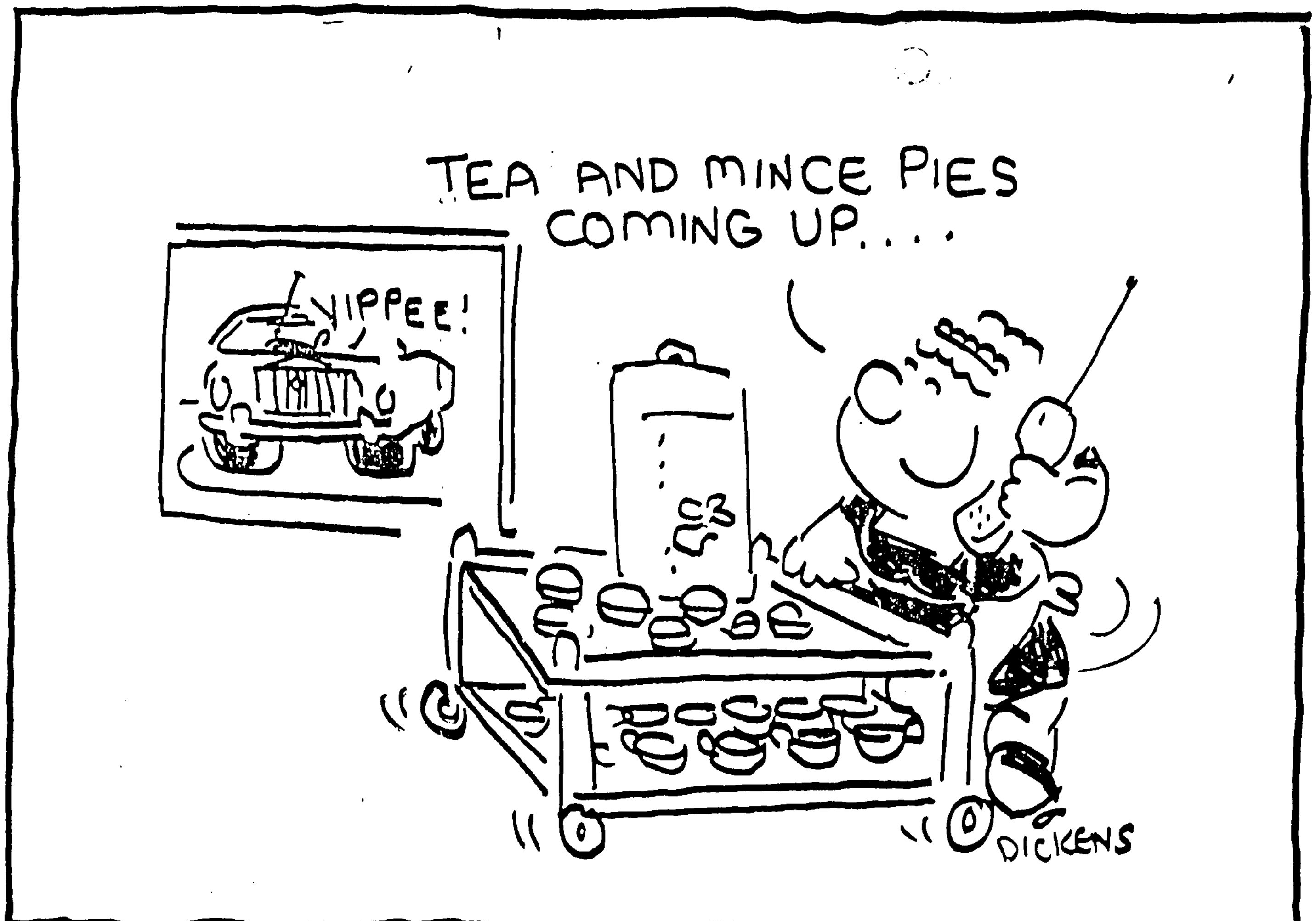
- NON-ALGEBRAIC DECODING
- SOFT-DECISION DECODING
- CODED MODULATION

SOFT-DECISION DECODING OF RS CODES

The most effective decoding algorithms for cyclic codes are those, which exploit the code structure and symmetry to the fullest possible extent, are extendable to soft-decision decoding and can offer trade-off between decoding performance and complexity. Minimum weight decoding seems to fulfill these requirements. It is based on the error trapping technique, which is the simplest of the syndrome decoding methods, combined with trial-and-error (systematic search) and step-by-step decoding methods, resulting a very effective reduced-search bounded minimum-distance decoding algorithm. It makes full use of the nature of a cyclic code and since it is essentially a form of minimum-distance decoding, its adaption for soft-decision decoding is quite simple, only requiring changing the metrics to soft (Euclidean) distance from the hard (Hamming) distance. Its complexity is proportional to the square of the block length or less making it very attractive for decoding short and medium length codes. Minimum weight decoding, a burst-and-random-error-correcting algorithm, is also an interpretative and unifying concept which explains the fundamental nature of syndrome decoding methods, and relates apparently distinct algorithms within a coherent framework. It manifests a practical example of an implementation of step-by-step decoding. In the trial-and-error stage of the decoding algorithm there exists an upper bound on the maximum number of trial-and-error positions necessary to be tested (i.e., not all trial-and-error positions need to be tried) which reduces the deocding delay (or latency) and overall complexity quite considerably.

A study of the extension of hard- and soft-decision minimum weight decoding to multi-level cyclic Reed-Solomon codes reveals that if the trial-and-error stage of the basic decoding algorithm is modified, then for example, with the RS (15,9) code, up to 1dB in the hard-decision case and 2dB in the soft-decision case is gained, when bit errors, rather than symbol errors, are corrected. Here the decoding algorithm can offer trade-off between complexity and performance. For example, in the soft-decision case the trial-and-error stage can be much simplified, and therefore complexity reduced, at the price of 1dB loss in the coding gain. The minimum weight decoding algorithm also can be extended to multi-level signalling schemes, such as ASK, m-ary PSK and QAM. On the two-dimensional 16-QAM AWGN channel, for example, simulation results show that the soft-decision minimum weight decoding of RS (15,9) code offers 1.5dB extra coding gain over the hard-decision case and an overall coding gain of 2.5dB. This combined coding and modulation scheme has very low complexity.

PGFAM25.VW



MOBILE RADIO

CYCLIC AND CONV. CODES

RS CODES

- MINIMUM WEIGHT DECODING

  (CONTINUED DIVISION
  ALGORITHM ≡ ERROR TRAPPING)

- HARD-DECISION RS MWD

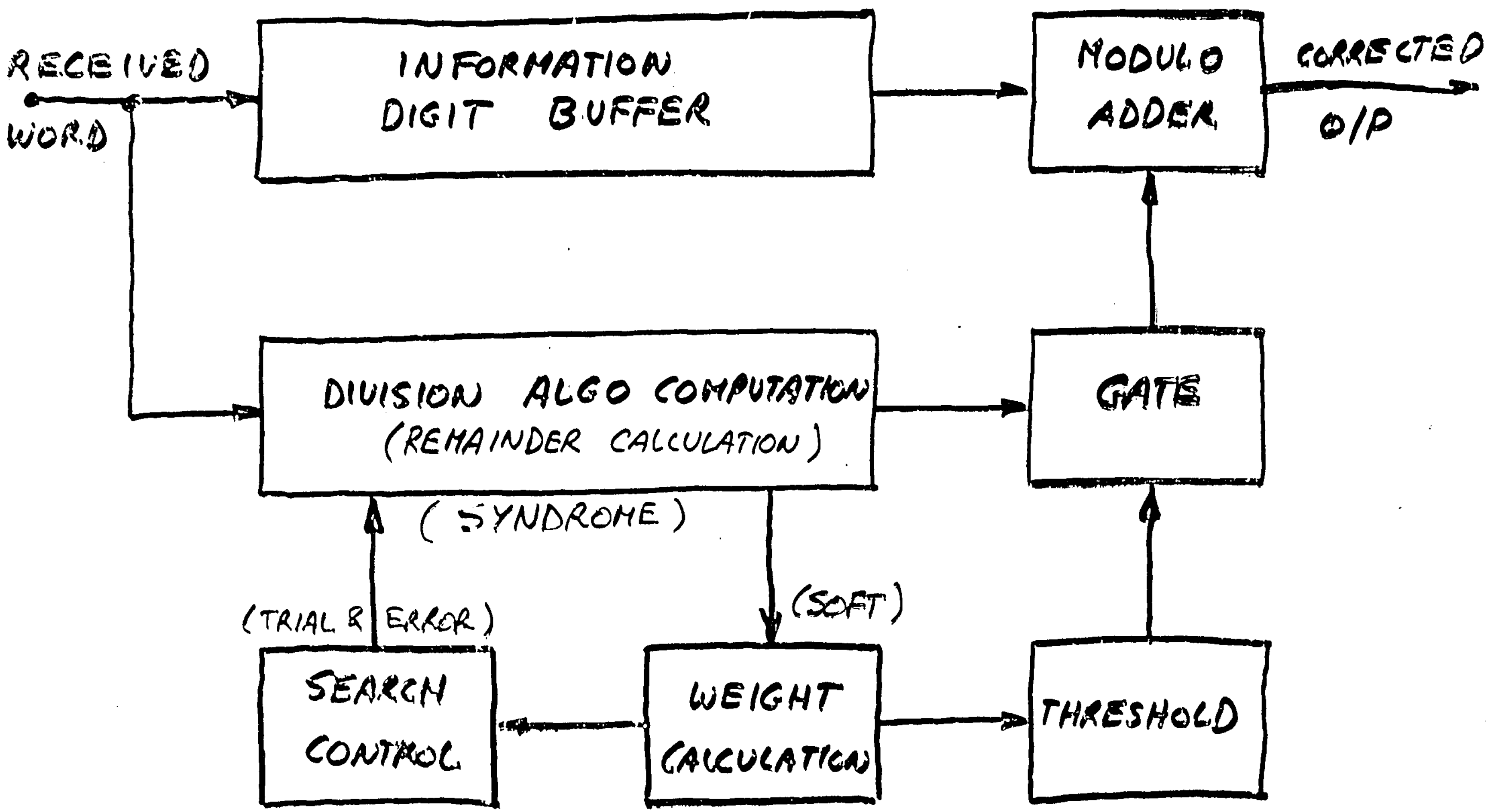
  SYMBOL DECODING
  BIT DECODING

- SOFT-DECISION RS MWD

  BINARY SIGNALLING
  ASK "
  QAM "

- SOFT-DECISION RS MLSD/MWD

  QAM SIGNALLING



MINIMUM WEIGHT DECODER

THERE IS A YOUNG FELLOW OF BONAS,

WHO WANTS TO USE CYCLIC DECODERS;

WHEN HE TRIES SOFT-DECISION,

FOR BETTER PRECISION;

THE GAIN TO HIS SYSTEM IS A BONUS

HARD SYMBOL MWD

RS (7, 3, 5) $t = 2$ $m = 3$

$G(x) \equiv 1\ \alpha^4\ \alpha^2\ \alpha^4\ 1$

1	1	1	1	1	1	1	$X(x)$
1	0	0	1	0	0	0	$E(x)$
0	1	1	0	1	1	1	$Y(x)$
	1	α^4	α^2	α^4	1		$G(x)$
0	0	α^5	α^2	α^5	0	1	$w = 4$
		α^5	α^2	1	α^2	α^5	
0	0	0	0	α^4	α^2	α^4	3
α	α^4			α^4	α	α^6	
α	α^4	0	0	0	α^4	α^3	4
α^6	α	α^4			α^4	α	
α^5	α^2	α^4	0	0	0	1	4
α^4	α^2	α^4	1			1	
1	0	0	1	0	0	0	$w = 2$: ERRORS

0		000
1		001
α		010
α^2		100
α^3	$= \alpha + 1$	011
α^4	$= \alpha^2 + \alpha$	110
α^5	$= \alpha^3 + \alpha^2 = \alpha^2 + \alpha + 1$	111
α^6	$= \alpha^3 + \alpha^2 + \alpha = \alpha^2 + 1$	101

HARD BIT MWD

001	001	001	001	001	001	001	$X(x)$
001	000	000	001	000	000	000	$E(x)$
000	001	001	000	001	001	001	$Y(x)$
	001	110	100	110	001		$G(x)$
000	000	111	100	111	000	001	$w_{bit} = 8$
111	100	110	000	000	000	001	$w_{bit} = 7$
110	100	110	001			001	
001	000	000	001	000	000	000	$w_{bit} = 2$, ERRORS

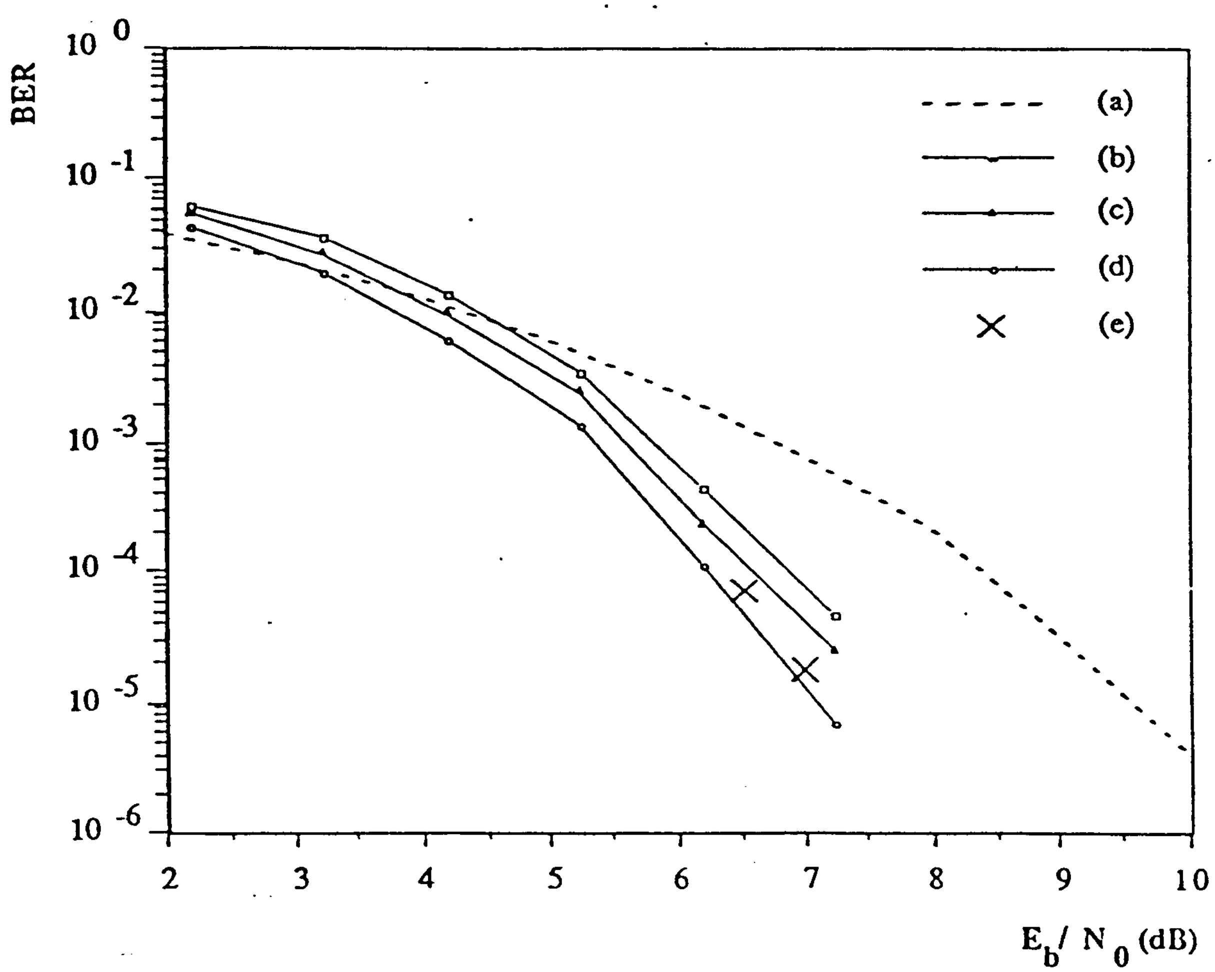
FOR 2 SYMBOL ERRORS :-

$$2 \leq w_{bit} \leq 6$$

WITH THE RS(15, 9, 7) $m=4$, $t = 3$ CODE

$$3 \leq w_{bit} \leq 12$$

AND OPTIMUM THRESHOLD IS FOUND TO BE 4



Hard-decision MWD Performance of the RS (15, 9) Code.

(a) uncoded, (b) $t = 3$ *(symbol decoding)*, (c) $t_{bit} = 4$ *(bit decoding)*,

(d) $t_{bit} = 4$ *(bit decoding with modified trial-and-error decoding)*,

(c) hard-decision maximum-likelihood *(symbol)* decoding (calculated).

TRIAL & ERROR:

IF INITIAL TRAPPING FAILS, CONVERT EACH RECEIVED SYMBOL TO ALL OTHER SYMBOLS, IN TURN, ATTEMPTING TO TRAP AFTER EACH CONVERSION

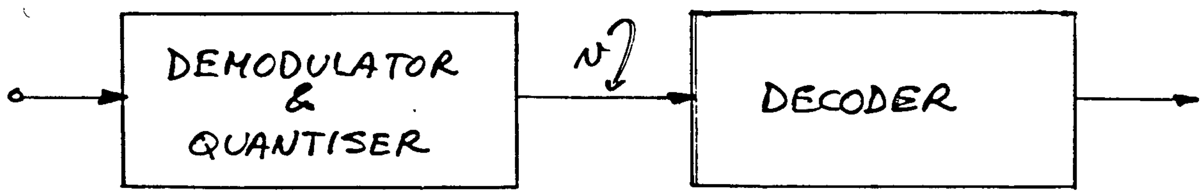
MODIFIED (EXTENDED) TRIAL & ERROR:

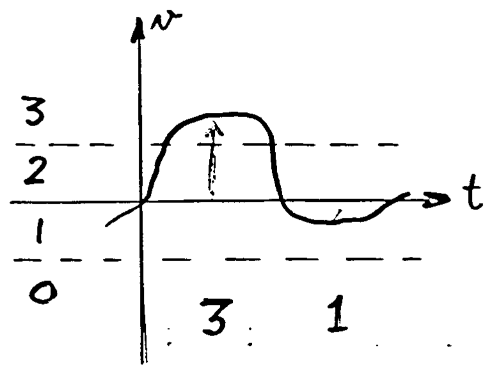
IF ABOVE FAILS, CONVERT ALL PAIRS OF RECEIVED SYMBOLS TO SYMBOLS ONE BIT AWAY, IN TURN, AND ATTEMPT TO TRAP

MODIFIED AND SIMPLIFIED TRIAL & ERROR:

AS FOR MODIFIED CASE, BUT INITIAL TRIAL & ERROR CONVERSION IS ONLY TO SYMBOLS ONE AND TWO BITS AWAY

SOFT DECISION DECODING





Q = NUMBER OF QUANTISATION REGIONS (= 4)

SOFT DISTANCE :

BETWEEN DIGITS $= |v_i - v_j|$

BETWEEN WORDS $= \sum_{l=1}^{n} |v_{il} - v_{jl}|$

EXAMPLE

$$\begin{array}{r} 3102 \\ 1230 \\ \hline 2132 \end{array}$$

$\Rightarrow 8$

SOFT BIT MWD (BINARY SIGNALS)

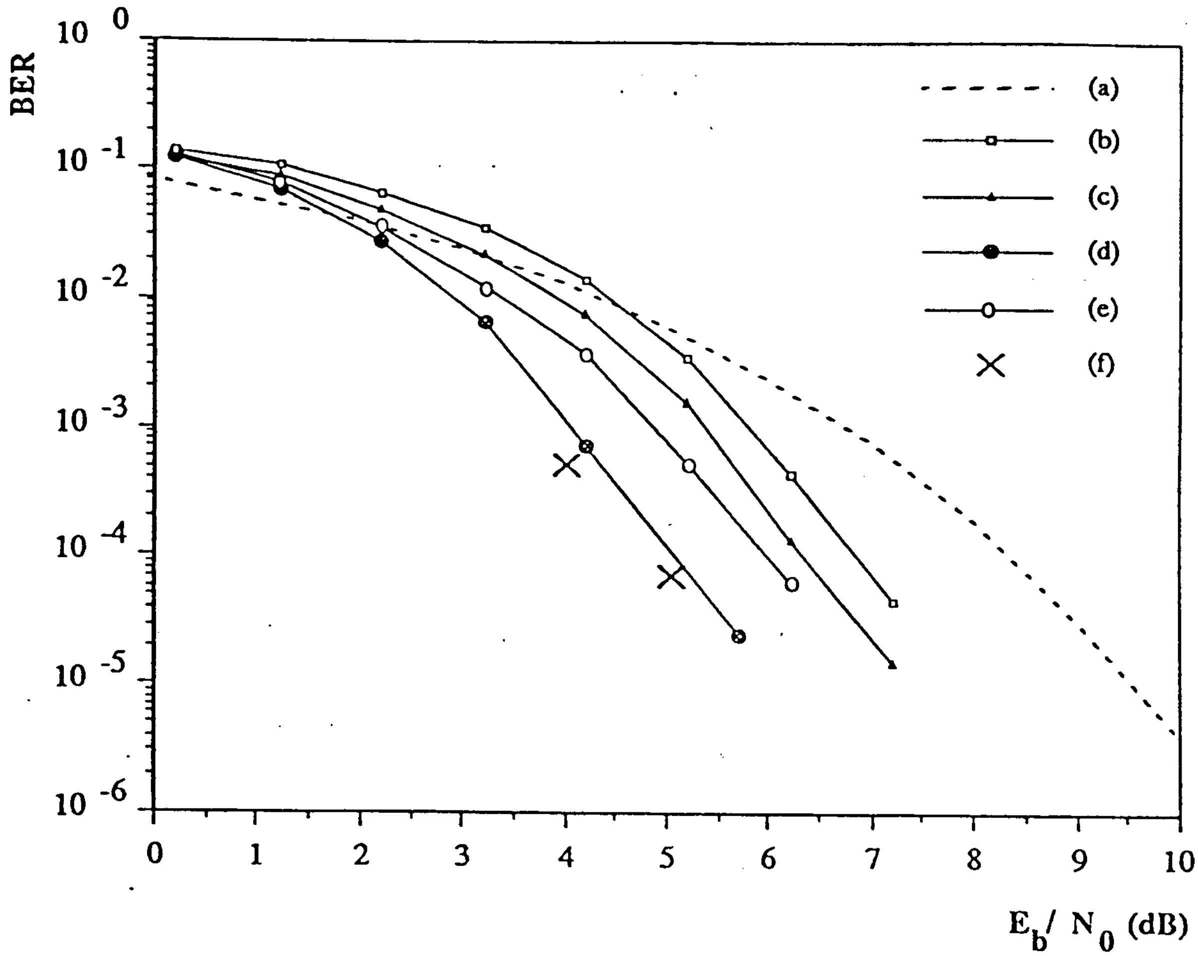
$$t_s = \left\lfloor \frac{d(Q-1)-1}{2} \right\rfloor = 17 \quad \text{FOR } Q = 8$$

007	007	007	007	007	007	007	$X(x)$	
007	000	000	005	020	000	000	$E(x)$	
000	007	007	002	027	007	007	$Y(x)$	
	007	770	700	770	007		$G(x)$	
000	000	777	702	757	000	007	w_{sbit} =	56
		777	700	007	700	777	$\alpha^5 G(x)$	
000	000	000	002	750	700	770		33
070	770			770	070	707	$\alpha^4 G(x)$	
070	770	000	002	020	770	077		53
707	070	770			770	070	$\alpha^4 G(x)$	
777	700	770	002	020	000	007		53
770	700	770	007			007	$G(x)$	
007	000	000	005	020	000	000		14 ERRORS

FOR RS(15, 9, 7) $m = 4$ AND $Q = 4$

$$12 \leq w_{sbit} \leq 84$$

OPTIMUM THRESHOLD IS **42**



Soft-decision MWD Performance of the RS (15,9) Code.

(a) uncoded, (b) $t = 3$ *(hard symbol decoding)*,

(c) $t_{s\text{-}bit} = 42$ *(soft bit decoding)*,

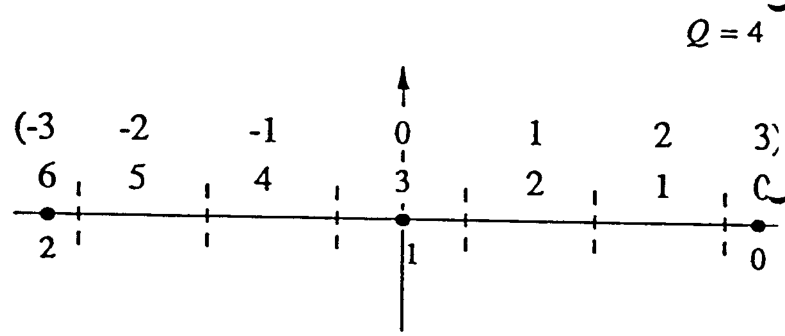
(d) $t_{s\text{-}bit} = 42$ *(modified trial-and-error decoding)*,

(e) $t_{s\text{-}bit} = 42$ *(modified & simplified trial-and-error decoding)*,

(f) soft-decision maximum-likelihood *(symbol)* decoding (calculated).

SOFT-DECISION WITH ASK SIGNALLING

high confidence	2	→	6
low confidence	2	→	5
low confidence	1	→	4
high confidence	1	→	3
low confidence	1	→	2
low confidence	0	→	1
high confidence	0	→	0



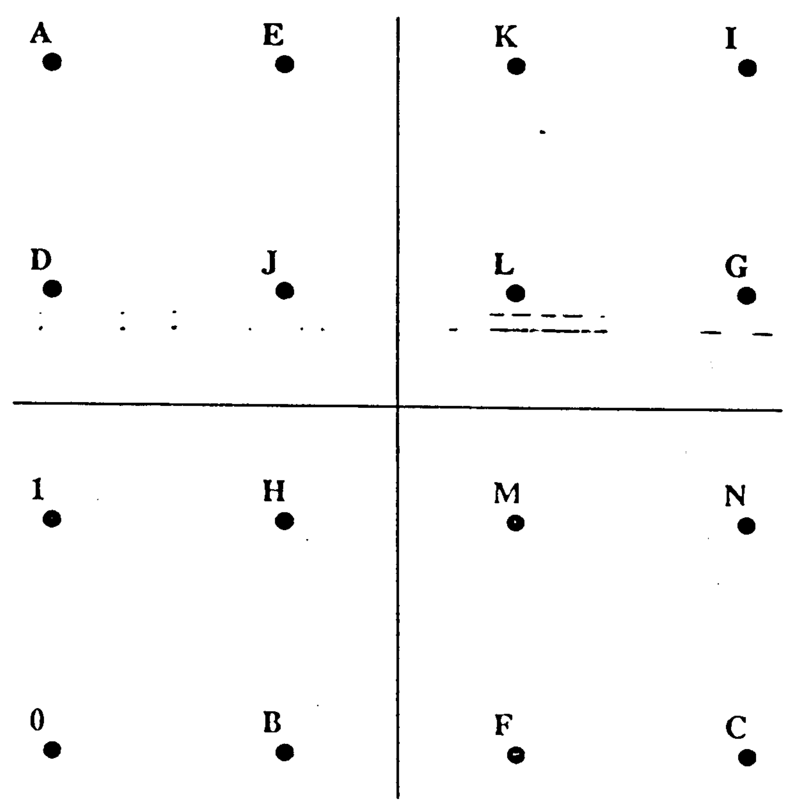
RS (3, 2, 2) IN GF(3)

$$G(x) = x + 2 \equiv 12$$

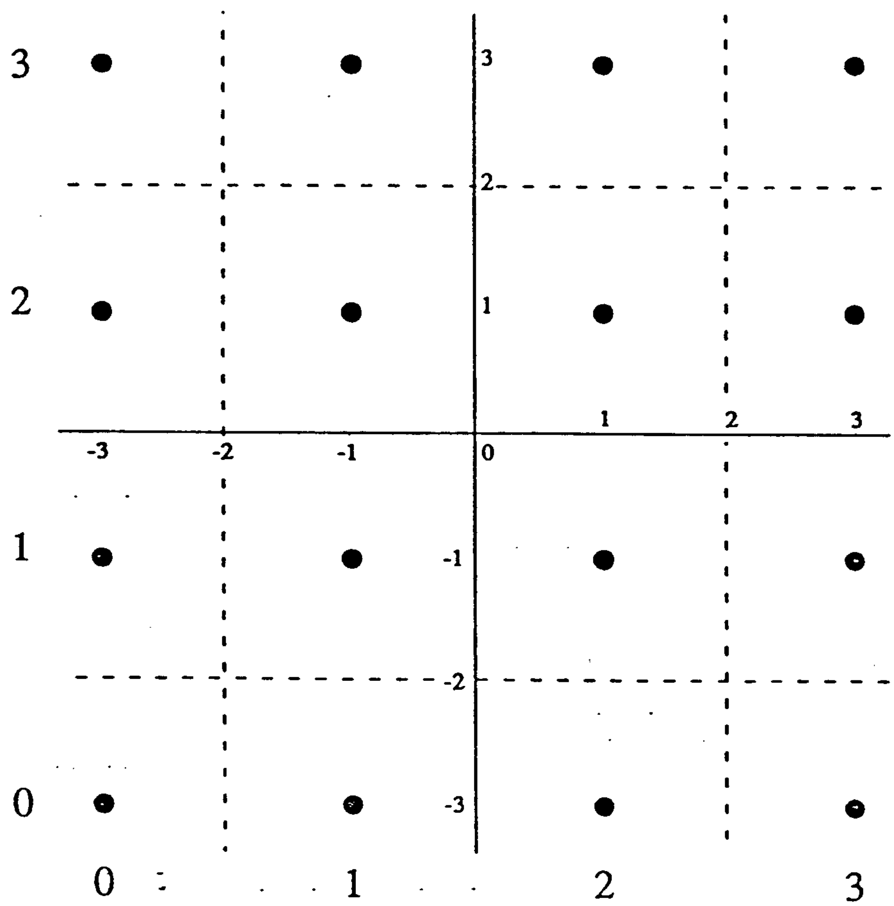
$$t_s = \left\lfloor \frac{d(Q-1)-1}{2} \right\rfloor = 2$$

	SOFT WT.	
6 6 6	↓	X
0 2 0		E
6 4 6	16	Y
6 3		2.G
0 1 6	7	(S)
3 6		2.G
3 1 0	4	
3 6		G
0 5 0	5	
6 3		2.G
0 1 3	4	
6 3		2.G
6 1 0	7	
6 3		2.G
0 2 0	2	SOFT ERRORS

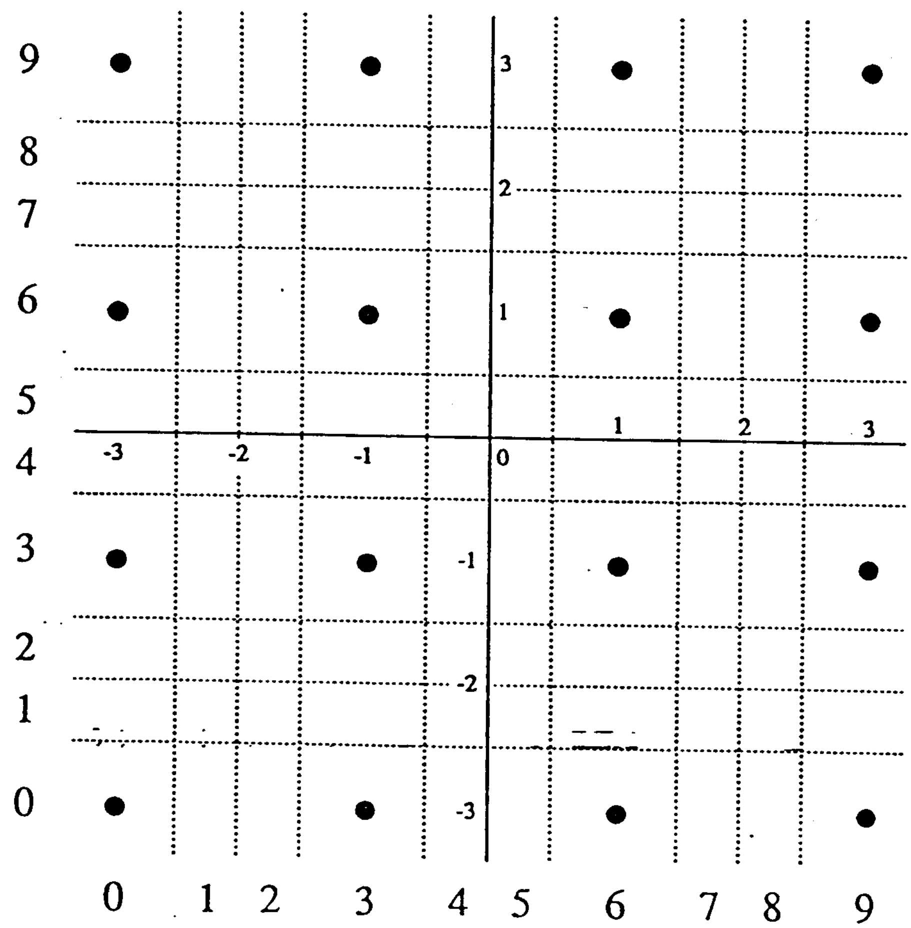
symbol	as a 4-tuple
0	0 0 0 0
1	0 0 0 1
A	0 0 1 0
B	0 1 0 0
C	1 0 0 0
D	0 0 1 1
E	0 1 1 0
F	1 1 0 0
G	1 0 1 1
H	0 1 0 1
I	1 0 1 0
J	0 1 1 1
K	1 1 1 0
L	1 1 1 1
M	1 1 0 1
N	1 0 0 1



Two–dimensional Gray mapped 16-QAM signal constellation.



Hard-decision quantisation thresholds and corresponding confidence values on the 16–QAM channel.



Soft-decision quantisation thresholds and corresponding confidence valu[e]
on the 16–QAM channel.

$Q = 4$

The MWD algorithm in the two-dimensional QAM channel, for example, would consist of the following steps :

Let the transmitter codeword be **c**, and the received signal be **r** at the decoder;

Divide **r** by the first finite field subtracter (*i.e.*, the appropriate shift of the generator sequence, $G(x)$, multiplied by the appropriate field element; using the corresponding decision symbol in the soft-decision case) thus forming a syndrome (the first finite field subtracter is also the first candidate codeword);

(*) Calculate the Euclidean distances between the symbols of **r** and the symbols of the candidate codeword, where the Euclidean distance is the root of the sum of the squares of the metrics in each dimension;

Calculate the "weight", w_e, by adding up the Euclidean distances found in (*);

If $w_e \leq t_e$ (threshold determined experimentally for HD and SD)
  the candidate codeword is taken as the decoded codeword;
  stop;

else Shift $G(x)$ cyclically, and find the next corresponding finite field subtracter with respect to the previous syndrome;

Divide **r** by the sum of (all) subtracters found so far, *i.e.*, by the new candidate codeword, thus forming a new syndrome;

Go to (*) and continue;

If after a total of $k + n$ shifts of $G(x)$, w_e has never fallen to t_e or less, convert each symbol of **r** in turn (in the soft-decision case, least confidence symbols first) to all other possible symbols in GF(q), and do the following in each case :

Divide r' (r with a converted symbol) by the first finite field subtracter to form a syndrome. The first finite field subtracter is also the first candidate codeword.

(*) Calculate the Euclidean distances between the symbols of r and the symbols of the candidate codeword;

Calculate the "weight", w_e, by adding up the Euclidean distances found in (*);

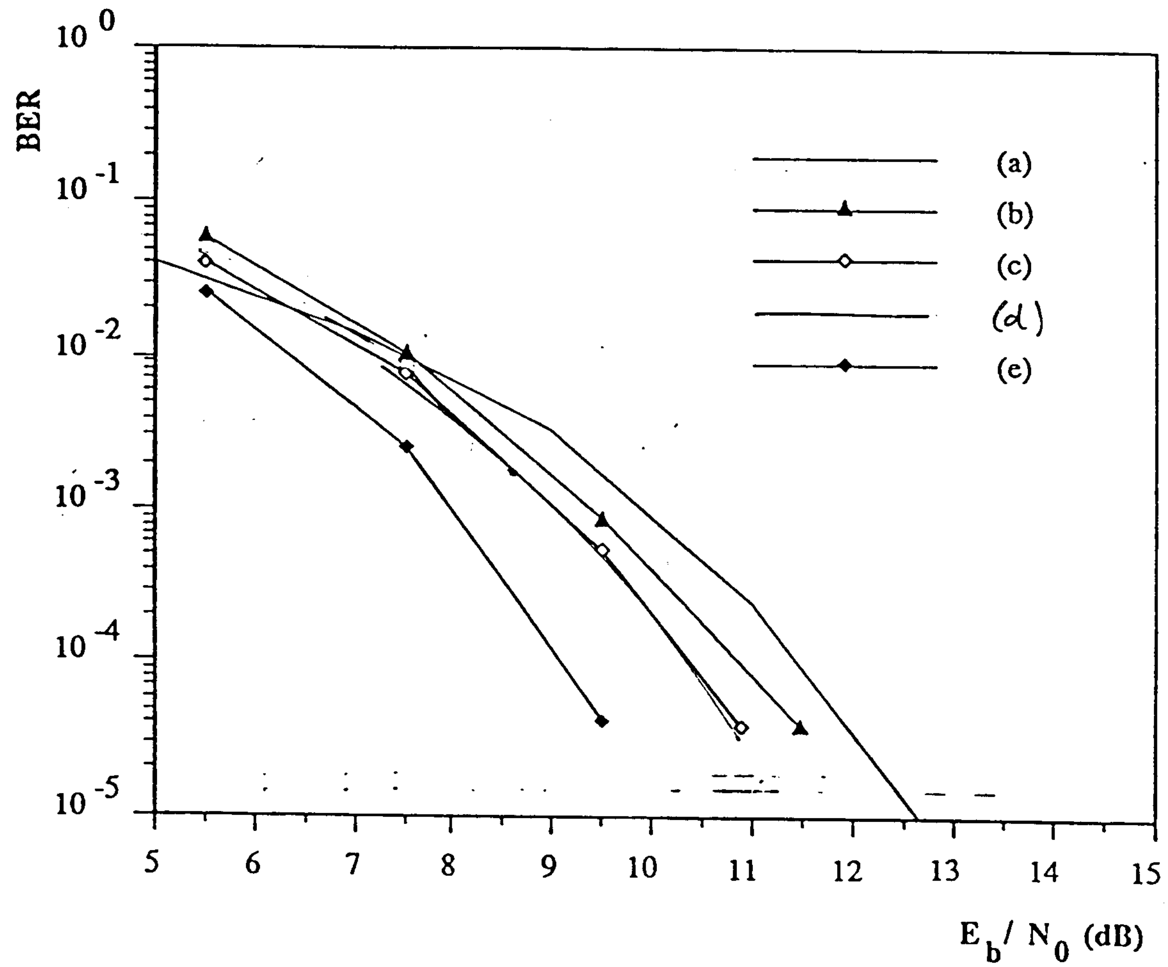
If $w_e \le t_e$ (threshold determined experimentally for HD and SD)
the candidate codeword is taken as the decoded codeword;
stop;

else Shift $G(x)$ cyclically (a total of $k + n$ times), and find the next corresponding finite field subtracter and syndrome after each shift;

Divide r' by the sum of (all) subtracters found so far, *i.e.*, by the new candidate codeword, thus forming a new syndrome;

Go to (*) and continue.

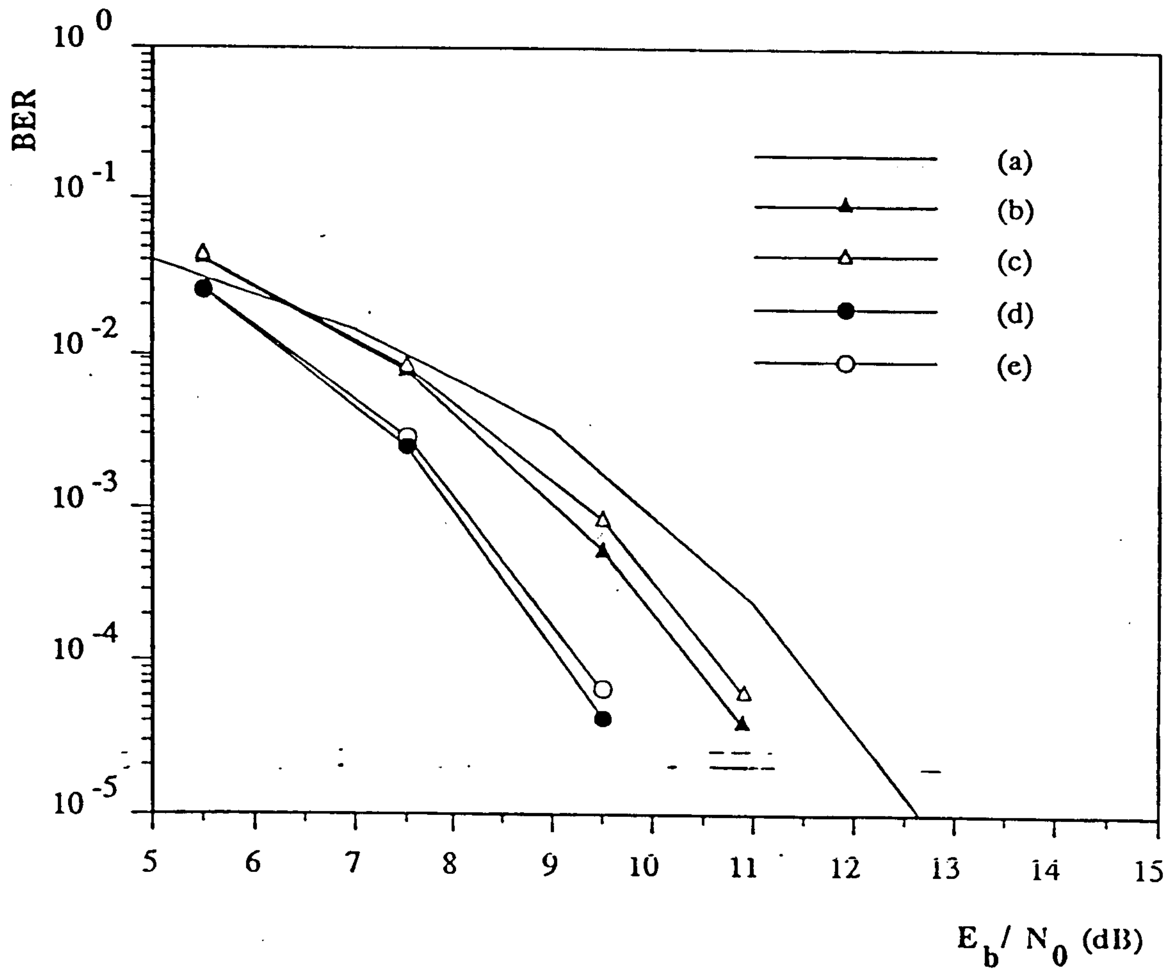
The algorithm stops when $w_e \le t_e$ is found, or after all n received symbols have been converted and all the steps for each r' have been performed, in which case no correction is made.



MWD performance of RS (15,9) code on the 16-QAM channel.

(a) uncoded, (b) $t = 3$ *(hard symbol decoding)*,
(c)* $t_{e\text{-}hard} = 4$, (d) RS(15,11) $t_{e\text{-}soft} = 14$ (e)* $t_{e\text{-}soft} \cong 16$.

* HARD EUCLIDEAN DISTANCE 1 OR $\sqrt{2}$
IN TRIAL & ERROR STAGE



MWD performance of RS (15,9) code on the 16-QAM channel

(a) uncoded, (b) $t_{e\text{-}hard} = 4$, (c) $t_{e\text{-}hard} = 4$ *(simplified trial-and-error decoding)*, (d) $t_{e\text{-}soft} \cong 16$, (e) $t_{e\text{-}soft} \cong 16$ *(simplified trial-and-error decoding)*.

HD EUCLIDEAN DISTANCE 1
IN TRIAL & ERROR

MAXIMUM LIKELIHOOD SEQUENCE DECODING (MLSD) ALGORITHM (TAIT, DORSCH)

BLOCK CODE $\mathbf{C}$
TRANSMITTED WORD c
RECEIVED VECTOR r

GENERATE SEQUENCE OF VECTORS $x_0, x_1, \ldots$ WHERE x_i IS i^{th} MOST LIKELY ESTIMATE OF c; ie :

$$p(r/x_0) \geq p(r/x_1) \geq \ldots$$

DECODING IS COMPLETE WHEN $x_i \in \mathbf{C}$

COMPLEXITY DEPENDENT ON COVERING RADIUS OF CODE (RADIUS OF SPHERE GUARANTEED TO ENCLOSE A CODE WORD, CENTRED AT r)

$$(q-1)\sum_{i=0}^{s} \binom{M}{i} \text{ ITERATIONS}$$

GUIDED BLIND WALK AMONG SPARSE CODE WORDS!

COMBINED MLSD AND BOUNDED DISTANCE DECODING (eg MWD).

GUIDED SIGHTED WALK !

MWD WITH $0 \leq e \leq t$ ERROR CORRECTION

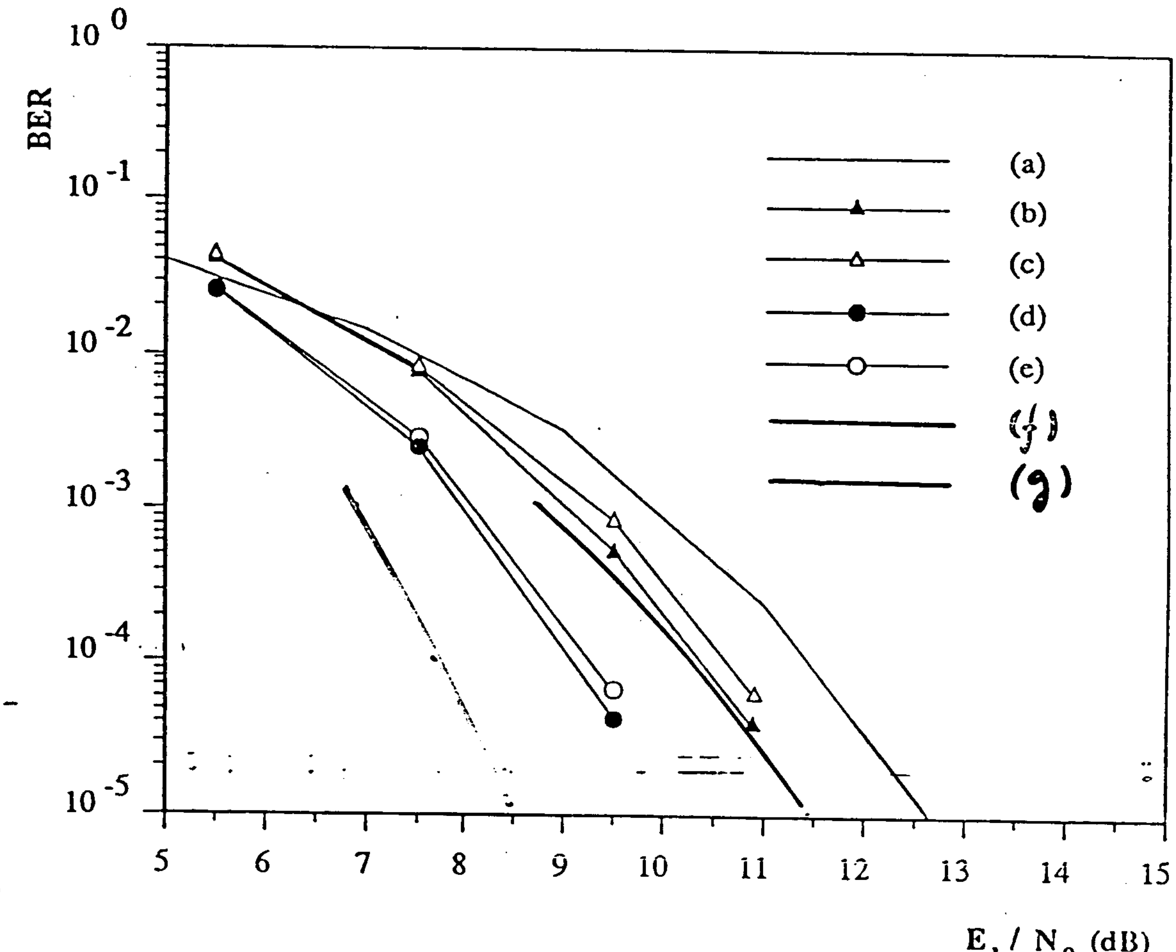
MLD IF $d\{x_i, r\} \leq t$

OR O/P BEST x_i AFTER FIXED NUMBER (LARGE) OF ITERATIONS

$\equiv$ MLSD IF $e = 0$

COMPLEXITY $\propto 1/e$

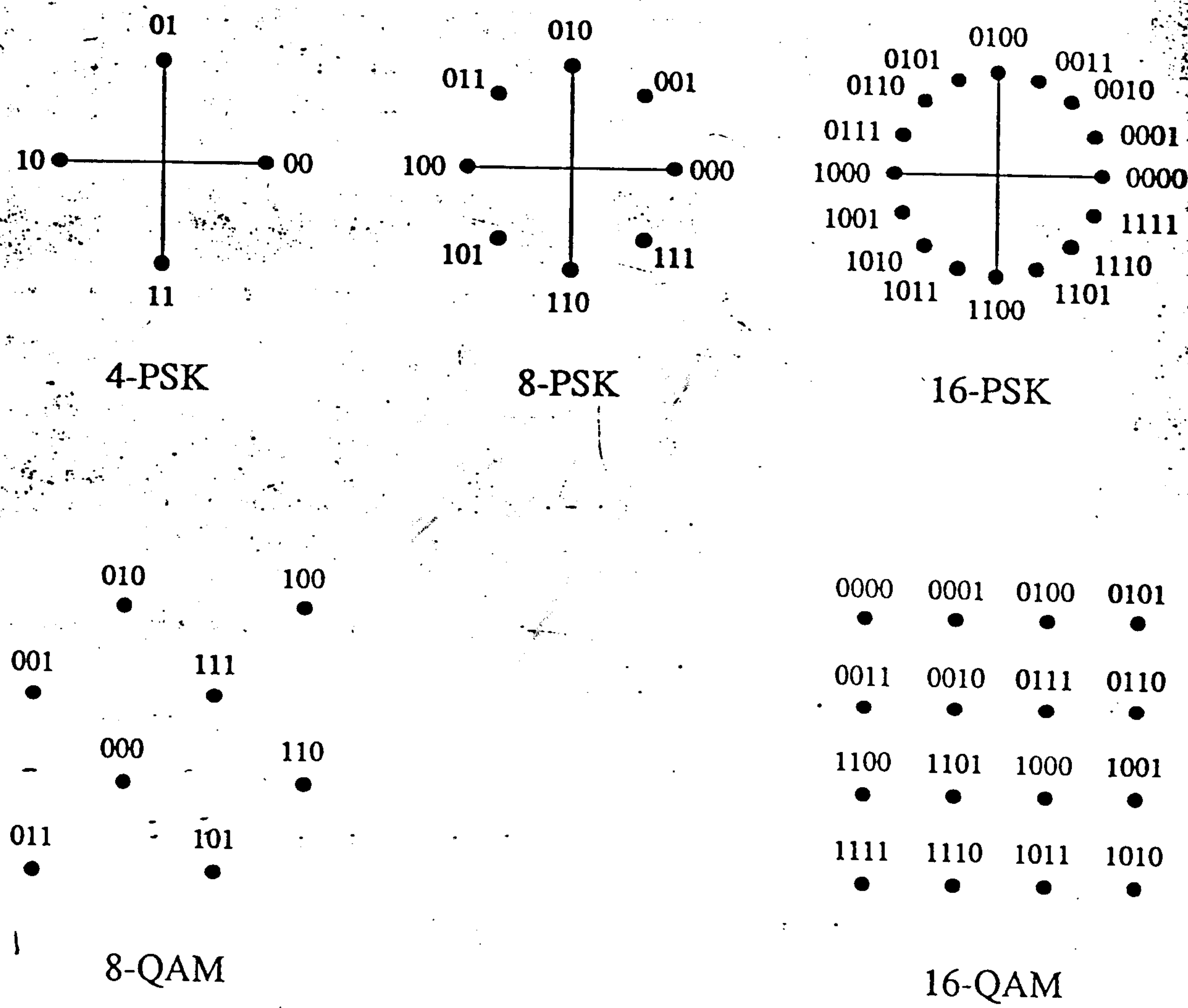
CAN BE ADAPTED FOR ANY SD METRIC, SUCH AS EUCLIDEAN DISTANCE IN A MULTI-LEVEL SIGNAL CONSTELLATION



MWD performance of RS (15,9) code on the 16-QAM channel.

(a) uncoded, (b) $t_{e\text{-}hard} = 4$, (c) $t_{e\text{-}hard} = 4$ *(simplified trial-and-error decoding)*, (d) $t_{e\text{-}soft} \cong 16$, (e) $t_{e\text{-}soft} \cong 16$ *(simplified trial-and-error decoding)*.

(f) MLSD / MWD (g) 8-QAM UNCODED



Signal Set Constellations